

On Wage Inflexibility

by

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Summary: I develop a wage setting theory starting with an imaginary flexible wage competitive economy with incomplete insurance markets hence both workers and firms are exposed to aggregate risks against which they cannot insure. A firm departs by offering jobs to regular employees according to a wage scale with fixed duration, typically four quarters, specifying a wage equaling the corresponding *expected flexible competitive wage*. At duration's end a new offer is made. The offer is formulated as a game in which the firm commits to make only expected wage offers. A firm can lay-off workers at any time. All risk averse workers accept such offer, it is optimal for the firm to make the offer and the equilibrium is perfect, explaining why wages do not fall in recessions. This result appears to conflict with the theorem stating convexity of profits in prices. The Theorem is incorrectly interpreted to claim expected profits rise with price volatility. Since prices and wages fluctuate due to shocks I show instead that a *profit function is concave in shocks* hence volatility reduces expected profits. An equilibrium is then an efficient mutual insurance of the firm and its employees in which the firm has a deep interest. When all firms depart, this institution gives rise to a mean wage of regular employees which reflects the staggered wage structure which emerges from the random distribution of the dates when job offers are renewed. It also results in random involuntary unemployment which is the social cost of this mutual insurance. Taking into account wage flexibility of irregular jobs, mean wage is a relatively inflexible function of inflation, of unemployment and of a distributed lag of past productivity.

JEL classification: D21, J2, J3, J6.

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There is probably no subject in Economics on which more has been written than wage determination and unemployment, yet the problem remains a challenge. It is thus wise to recognize, at the outset, that any new approach to the problem will include some topics that have already been analyzed by others and on these, the present work can only be a reformulation of earlier ideas. A “reformulation” does not preclude a drastic new perspective with different theoretical and policy implications. Foremost among these is the idea wages are sticky or inflexible, and that the labor market is not a regular competitive auction market (e.g. see Malcomson (1999) and citations there). But when it comes to explaining why wages are sticky then, apart from the literature on “implicit contracts,” most writings on the subject associate sticky wages with failure of competitive markets due to union contracting, monopsony power of labor and bargaining between firms and workers. Under such conditions the problem of unemployment is rather simple and leads naturally to the classical conclusion argued by generations of economists going back to Pigou’s (1927) discussion of British labor markets. If unemployment is caused by market imperfections then the problem of unemployment can be solved by restoring competition and wage flexibility. In this chapter I offer a different explanation of why wages are inflexible. It starts from the vast empirical evidence that union membership in the US is a negligible fraction of workers and has virtually no effect on wage setting. “Implicit contracting” did offer a different perspective and I evaluate it later.

The empirical evidence for inflexible wages is decisive. Yet, most work on competitive Growth Theory and Real Business Cycles assumes competitive labor markets with flexible wages and most New Keynesian Models (in short NKM) assume flexible wages hence full employment but sub-optimal output. Wage and price inflexibility are related as sticky wages contribute to price stickiness because inflexible marginal cost of rational firms reduce their motive to change prices. For this reason the extensive literature on staggered wage contracts (e.g. Fischer (1977), Taylor (1979),(1980), (1999), Gertler and Trigari (2009) and references) is often used as a paradigm for deducing price stickiness. This paper argues that one must distinguish between motives for sticky prices and motives for inflexible wages hence inflexible wages and inflexible prices are different phenomena.

¹ This paper is very preliminary and is circulated as a compliment to my paper entitled “Stabilizing Wage Policy.” Research on the material discussed here is on-going.

I start in the next section with a brief review of the literature on inflexible wages. It sets the stage for an explaining why wages are inflexible. Later I take up the problem of wage setting that uses the theory developed here as a guide to formulate a practical wage adjustment process which can be incorporated in any model.

1. The Literature on Wage Inflexibility and the Definition of Involuntary Unemployment

The literature of wage inflexibility is large and diverse sources of data were used to document lack of full wage flexibility as required by a competitive auction market for labor. It is thus impossible to review the entire literature and, instead, I review issues and results that relate to the work in this book. Let us then start with our daily experience. It shows that, at least for permanent employees, wage and salary rates (including benefits) are almost universally fixed for a period which is typically one year. Before the start of the following year, work performance reviews take place and result in new rates set for next year. Such a new rate can equal the old hence the duration of a fixed wage could be longer than one year. Had wages been determined in a competitive auction market then at each pay day an employer would read the current wage rates off an open market from which to deduce the entire scale of wages paid to each employee. Wage rates and incomes would change every pay period, resulting in much higher volatility of both wage incomes and corporate profits. The mirror image of this daily experience is the conflict between actual data on volatility and the predictions of competitive models. Two ratios are relevant here. Let $(\sigma_w, \sigma_y, \sigma_n)$ be standard deviations of the real wage, output per capita and total labor employed. U.S. data since 1947 show $(\sigma_w/\sigma_y) = 0.38$, $(\sigma_n/\sigma_y) = 0.99$ while simulated competitive labor market models with flexible wages typically imply $(\sigma_w/\sigma_y) > 1.0$, $(\sigma_n/\sigma_y) < 0.35$ (e.g. see Kurz, Piccillo and Wu (2013), Table 3). This means that models with flexible competitive wages generate a level of wage volatility that is *order of magnitude* higher than found in the data. It also shows the well recognized fact that *volatility of output is mostly associated with fluctuations in unemployment* not in wages.

Turning to other evidence, in the *General Theory* Keynes made the celebrated conjecture that wages exhibit a counter-cyclical behavior and thereby generate a debate that initially involved Dunlop (1938), (1939), (1941), (1944), Tarshis (1938), (1939), Keynes (1939), Ruggles (1940) and others. A very large empirical literature followed which examined the econometric properties of wages and wage adjustments, both aggregate as well as individual. Extensive and excellent surveys are by Abraham and Haltwinger (1995), Brandolini (1995), Tobin (1972) and in a recent overview of Pencavel (2013). It is not an overstatement to say that the conclusion of this vast volume of work is that wages do not exhibit a stable and consistent adjustment to cyclical variables over time: real wages are unresponsive to unemployment and if they do respond, the response rate is

very small. One stable pattern, observed by many, is formulated in Tobin's (1972) summary of the conference papers showing that in response to changes in inflation or expected inflation, nominal wages do not fully adjust *either up or down*. For any change of 1 percentage point in the inflation rate nominal wages change by 0.4 - 0.7 percentage points hence real wages respond to inflation with delay. In short, most empirical work show nominal wages exhibit stickiness both up and down.

Micro data files derived *from union contracts* was examined by Taylor (1983) and Cecchetti (1984). They show that most wage agreements cover between one and two years. Taylor (1983) finds that wage setting is non-synchronized so that in any one quarter 15% of union wages are adjusted and in a year about 40% adjust. Cecchetti (1984) finds that the average period between wage changes is 7 quarters in the 1950s and 1960s when inflation was low, but it fell to about four quarters in the 1970s during the high inflation period. This reveals that the duration of wage setting varies with market conditions and certainly varies with the rate of inflation.

Studying data *of non-union members*, Lebow, Stockton and Wascher (1995), McLaughlin (1994) and Card and Hyslop (1997) use individual wage data from Panel Study of Income Dynamics and show the results are similar to union members. These files report wage changes only once a year hence a conclusion that wages were fixed over a time interval is often deduced from the observation that any distribution of wage changes over a year has a significant spike at zero. Restricted by such data, Card and Hyslop (1997), supplement it with data from the U.S. Current Population Survey from 1979 to 1993. They report that, among those who do not change jobs, between 6 and 15 percent of workers experience no change of their nominal wages from one year to the next, suggesting a relatively small fraction of workers whose wages change less frequently than once per year. They also confirm the results of Cecchetti (1984) that the frequency of wage adjustment increases with inflation.

The question of upward vs. downward wage rigidity has attracted some work. Using the same wage data specified above Card and Hyslop (1997) find that 15%- 20% of workers reported nominal wage reductions from one year to the next but conclude that *downward wage rigidity is as common as upward wage rigidity*. They estimate that downward nominal rigidity could have held up real wage changes by at most 1 percentage point per year. This result explains why almost all empirical macroeconomic models of price and wage rigidity study *symmetric* wage inflexibility. In contrast, some recent work studied the effect of *asymmetric* downward nominal wage rigidity (e.g. Daly and Hobijn (2013) and references there) based on the same wage data discussed above. Daly and Hobijn (2013) update the Card and Hyslop (1997) data file and, comparing the distribution of wage changes in 2006 with the distribution in 2011, they note two facts. One, a high concentration of workers at a zero nominal wage change, and (ii) a larger percentage of zero wage change in 2011 than in 2006. This last result,

showing the distributions of wage changes in 2011 is more asymmetric than in 2006, is the basis for conclusions of *downward* nominal rigidity of wages, arrived by some authors. They observe that the larger proportion of zero changes in 2011 occurs, relative to 2006, together with a decline of the negative part of the distribution. The authors deduce from this that a sizeable fraction of “desired” negative wage changes in 2011 became “no wage change.” But this is not compelling. It ignores other possible explanations for this difference in the distributions. One direct but obvious explanation is the fact that during a recession firms lay off undesirable workers rather than employ them at a lower wage (see Bewley (1999)). Hence the above reasoning about downward wage rigidity is nothing but the observation that in a recession undesirable workers whose wage is viewed by the firm as too high, are laid off hence absent from the sample, while desirable workers keep their job and do not experience major declines in their wages. That is, a sample that contains only workers who held their job has a serious selection bias.

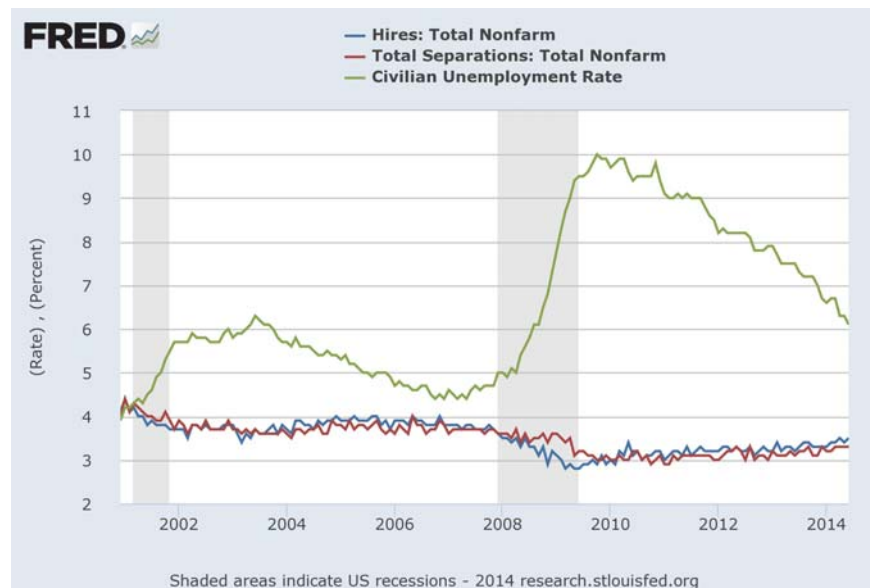
In sum, the evidence shows wages (including benefits) of most workers are typically adjusted at discrete intervals, mostly once a year. It also shows that adjustment of wages to inflation is incomplete and wage setting is independent of union membership. Wages clearly do not play the adjusting role to shocks that a price plays in a competitive auction model, and most aggregate adjustment in response to large shocks is carried out by large changes in employment, not wages. It is then a requirement of any convincing theory of inflexible wages that it explains *why the involuntary unemployed have no or little impact on wage setting*. It also needs to resolve the old debate whether there are any circumstances when involuntary unemployment can be cured by wage flexibility.

Many studies have incorporate inflexible wages and some recent ones (e.g. Shimer (2004), Hall (2005a), (2005b), Gertler and Trigari (2009), Blanchard and Galí (2010)) show that the introduction of ad hoc wage stickiness can account for some of the wage and employment volatility phenomena I noted earlier. Equally important is the fact that work such as Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007) conclude that wage inflexibility is needed to account for the observed data. I agree with the aims of these authors but do not accept their formalization of wage inflexibility, and will approach the problem differently. However, to place this work in context, there are two issues that need to be addressed. First, most recent work on wage setting is carried out in a job search and matching framework. Since my central concern is with *involuntary* unemployment, it would be clearer if I explain why ideas from search and matching are liberally employed when applicable but also why I do not formally adopt this framework here. Second, and that follows from the first, I give a formal definition of “involuntary” unemployment.

Search and matching theory was developed by Diamond(1982), Mortensen(1982) and Pissarides (1985).

It has been influential in explaining the natural rate of unemployment – an important labor market phenomenon. The theory postulates a rate of random separation from jobs, a rate of new job postings and a rate at which job openings are filled. Search cost, valuation of a job held by a worker and firm’s value of an employed worker , all imply equilibrium unemployment determined by factors such as matching technology, unemployment insurance, etc. Since at full employment separation rate equals the rate at which job opening are filled, the natural rate is an equilibrium rate of *voluntary* unemployment. The CBO estimates of the natural rate place it in a narrow range. Between 2007:4 and 2013:3 the natural rate fluctuated between 5% and 6% while actual unemployment rate was as low as 4.6% and as high as 15% if one includes discouraged workers during the Great Recession. The concern of the present study is involuntary unemployment relative to which a natural rate is essentially exogenous.

Figure 1



Another fact to consider is the high separation rate. Monthly US separation rates fluctuated around 3.8% in early 2000's and around 3.3% in 2014. Such high separation rates that exceed 45% annually question the significance of a job’s capital value for a worker or the value of an employee to a firm and suggest that other factors affect labor mobility. In fact, a recession is not a time when the separation rate rises dramatically hence some scholars treat it as an exogenous constant (e.g. Hall (2005b)). *Big* changes in unemployment are caused by *small* changes in firms’ recruiting efforts reflected in the rate of their new job postings with cost to firms that are

relatively small in practice. Figure 1 highlights this process and shows a decline of about 0.5% in the mean new hire rate during the one year 2008:7-2009:7 was sufficient to raise unemployment to an effective rate much higher than the standard measured rate of 10%. A small rise in the mean hire rate above separation initiated a slow decline in the unemployment rate since 2010. Also, the rate at which separated workers find jobs fluctuates between 30% and 60% *per month* hence most separated workers quickly find new jobs but high involuntary unemployment rates entail a rising fraction of workers who become long term unemployed with significant social cost. In short, the decisive factors altering involuntary unemployment *are changes in demand of firms for labor*, expressed in changed recruiting efforts. Finally, changes in unemployment which are due to factors associated with search technology are only mildly cyclical except for changes in job posted which is the demand for labor. As to the Nash Bargaining wage, it is unsatisfactory for two reasons. It is in conflict with the fact that wages are inflexible and second, it implies an efficient solution whereas involuntary unemployment must be considered an inefficient social outcome.

For the reasons outlined I conclude that a study of involuntary unemployment can assume the search process is exogenous since its machinery is unsuitable for the study of involuntary unemployment. Since all estimates show the natural rate exhibits very small fluctuations, these can be ignored if one is primarily concerned with the *large* swings of involuntary unemployment. In aggregate models with involuntary unemployment I then make the simple assumption that the natural rate is *a constant at 5%* thereby accepting the key conclusion of the search and matching theory. Indeed, one can further simplify by assuming the natural rate is a constant even at steady state and is attained when the supply of labor exceeds the demand by 5%. It then follows that involuntary unemployment is a percentage difference between the deviation of labor supply and the deviation of labor demand from steady state. Labor supply is derived from dynamic optimization and labor demand depends upon market structure. Starting in full employment with a sticky wage function, if firms' job postings decline, involuntary unemployment emerges. If job postings and the demand for labor rise, involuntary unemployment turns negative drawing some labor from those who are naturally searching. In that case the natural rate temporarily declines. The data shows that this is rare and over the last half century occurred only during the Korean and Viet Nam wars. I later point out firms have other options to hire irregular workers (defined later). I thus assume that at the given wage rate firms can hire the work force they desire hence are always on their demand function for labor. Search unemployment is, indeed, an important force that permits a firm to be on its demand function since the searching unemployed are willing to work for a posted wage and firms alter their recruiting effort when they need to change their labor force.

My definition of involuntary unemployment is thus *entirely traditional* and implies such unemployment exists only if the wage is not the one that makes workers employed equal total labor supplied, accounting for natural unemployment. Also, *all else being equal*, at a lower wage any firm prefers to hire more workers. This requires me to address the standard question of how can a sticky wage exist in a market with rational actors and also the classical question of when wage flexibility could eliminate unemployment.

2. Formulating a Game of Expected Wage Offers

To investigate why wages are inflexible I examine how firms and workers respond to uncertain future wages in an incomplete competitive auction market. *Incomplete* means there are no markets for contingent claims to enable workers ensure their future wages against aggregate shocks or firms hedge their future wage cost. In short, workers and firms cannot insure against aggregate shocks: a complete Arrow-Debreu structure is only a conceptual reference. Hence, consider an *imaginary* economy with flexible wages in a competitive labor market and refer to it as the “flexible wage market” where $\{W_t \equiv (W_t^N/P_t), t = 1, 2, \dots\}$ is an equilibrium stochastic process of real wages. For well known reasons it is impossible for a worker to use anticipated future wage income as collateral for bank loans hence a fluctuating wage rate is a difficulty for both workers and firms. Workers find it hard to smooth consumption over time but high wage volatility also contributes to profit volatility hence to higher riskiness of investments in a firm and lower firm valuation. An important factor in this context is the convexity of the firm’s profits with respect to wages and prices which proposes the common view that a firm benefits from fluctuations, a crucial topic discussed later. But then, *could other market structures emerge to replace the competitive, flexible, auction labor market?*

To explore other institutions I expand the strategy space to allow a firm and workers to deviate from the flexible wage market by making and accepting/rejecting job offers with less risky wages. The allowable strategies are detailed in the next section but their aim is restricted to a *mutual hedging against future risky wage fluctuations*. Offers may include intentions or promises of future actions but enforcement must result from equilibrium perfection rather than from the use of economic power.

There are two main groups of workers who receive job offers discussed in the next Section. *Most are the firm’s current regular employees* who receive a work performance evaluation followed by an offer of continued employment. The second group consists of a firm’s new hires for “regular” jobs who are assessed as qualified and may engage in horizontal job change. These may be phased in via a “trial” period before being assigned a regular status. Such new hires then trigger work evaluations, regular employment and updated nominal wage offers hence

wages of new hires also adjust to inflation faster than wages of current regular firm's employees. This is significant since monthly turnover rate in the US fluctuated around 3.8% in early 2000's and around 3.3% in 2014. This rate exceeds 45% annually and all such new hires for regular jobs and horizontal job changes receive automatic evaluation with updated nominal wage offers.

There are, however, significant components of the labor force that do not fit the "regular" categories and it is important to keep them in mind since a large fraction of unemployed workers come from these groups:

1. *Entry level jobs*. These are typically viewed as temporary until qualified via work performance evaluation following which they may be turned into regular employees;
2. Most part time jobs whose holders are not considered regular employees;
3. Seasonal jobs in sectors such as construction, agriculture, hospitality, entertainment etc;
4. Jobs on large projects with short duration: oil drilling, aviation, military contracts, census takers etc;
5. Short term personal service providers such as consultants, researchers, information technology etc.

What is common to the jobs listed is that offers for them are made on a temporary but more frequent basis than those made to regular long term employees and without a long term relation between firms and workers. Wage rates for these jobs are more flexible and more responsive to current conditions. This fact will require an adjustment in the theory proposed below. I will refer to all these job categories as "non regular."

The discussion above shows a mean wage incorporates not only different workers but also diverse wage setting mechanisms. Since I end up using only one wage rate, it is important to see that when discussing the wage setting of "regular" or "irregular" jobs, I do not discuss jobs with different tasks that distinguish an engineer from a cleaning worker. Instead, two different jobs need to be interpreted *as jobs with the same tasks but with different working conditions along the lines listed in the previous paragraph*. Thus all workers will perform the same task but some are hired on a regular basis, others are temporary or seasonally employed.

2.1 "Expected Wage" Offers

I now explain a class of "Expected Wage" job offers made by a firm that hypothetically deviates from a currently active competitive flexible wage market. The experiment conducted is then imaginary, examining the stability of the flexible auction labor market. Under flexible wages all face uncertainty at date t of date $t+1$ wage. Now suppose a firm offers workers at t to work at $t+1$ for a known real wage $E_t[W_{t+1}|I_t] \equiv W_t^e$ which is date t *conditionally expected $t+1$ real wage*. The term "wage scale" will refer to a list $(W_t^{e1}, W_t^{e2}, \dots, W_t^{eJ})$ of wage functions that specify the wage of each of the J different job categories in the firm. Each element is a function of

state variables and specifies how wage offers change when conditions change. In a stationary economy the wage scale is fixed. Also, typically the wage remains constant for the duration of the offer after which it is revised in accord with the scale. The specifics of such offers are as follows:

1. the wage is based on a wage scale that depend on job description, responsibility, seniority, etc;
2. a new offer is made after a job review, at the end of the offer's duration. Existing employees have priority to receive offers for continued employment;
3. continued employment entails an opportunity for future promotions along the scale;
4. the firm retains the right to lay off workers at any time;
5. the firm commits to make only expected wage offers and never make lower offers that aim is to replace regular employees when the flexible wage market is lower.

This offer is a zero cost *mutual insurance* against wage risks and a layoff means a worker must seek an alternate offer or accept a job in the flexible wage market *where jobs are always available*. Also, the offer is not a contract that guaranteed either consumption or employment for the specified duration but it carries an important provision that grants existing employees priority to receive renewed offers at the end of the specified period of the job.

The offer may appear similar to an Implicit Contract but this is incorrect for several reasons. First, it is not a complete contingent contract since it covers only wages with no other benefits or cost. Moreover, it is a *mutual hedging arrangement* that will be prove to be beneficial both to the firm as well as to the workers. Second, an expected wage offer is Pareto improving when only one firm deviates from the flexible wage market. However, when all firms adopt the same strategies they give rise to an inefficient institution of involuntary unemployment where *firms and employed workers benefit at the expense of the unemployed who emerge as the social cost of the institution*: involuntary unemployment is not Pareto Optimal. Third, the implied equilibrium aggregates of expected wage offers are drastically different from levels of output, inflation and involuntary unemployment of the flexible wage competitive markets.

Several immediate issues are raised by the expected wage offer. The first is related to the *firm's wage scale or ladder* that is a very important part of a wage offer for several reasons:

(i) The internal pay scale and the firm's culture enable workers who receives wage offers to form expectations about future promotions and wage increases. Due to the wage scale *individual wages continue to rise even when mean wage of the firm is constant* because internal promotions and wage increases are induced by departures and retirements of more senior and higher paid workers. *The internal wage escalator within the firm is the basis of individual expectation for future wage changes.*

(ii) Most wage offers are made to existing employees at the time of work review who are also given priority to receive such offers if equally qualified.

(iii) A firm's wage scale is the key tool of labor market competition across firms. Since the wage scale informs workers about expected future wages, a firm competes for good quality workers by offering not only current wage but an entire scale. Adjusting a wage scale is the most potent competitive tool a firm uses in the labor market.

Three other factors have effects on the distributed lag structure of mean wage. First, although most wage offers are constant for a year, evidence from the Panel Study of Income Dynamics (e.g. Card and Hyslop (1997) , Daly and Hobijn (2013)) shows that a significant fraction of workers do not receive a wage increase at the time of work review. Hence, although a wage may formally be fixed for a year, in the data it could be fixed for a longer period. A complicating effect, stressed by Taylor (1979), (1999) and others, results from the fact wage offers are unsynchronized across firms. This implies that at any date there is a *distribution* of effective wages across firms for the same job, depending upon past dates when these wages were set. Third, and this works in the *opposite direction*, the wage distribution is drastically restricted by the very high turnover rate that exceeds 45% annually and which significantly updates wage setting for the obvious reason explained earlier. This reduction in the lag structure comes in addition to the fact that wage rates paid for "irregular" jobs are updated every quarter further reducing the lag structure of mean wage.

My model of a firm's behavior is inspired mostly by empirical evidence provided in the important work of Bewley (1999) who reports several empirical regularities that need be kept in mind. First, notwithstanding the title of his book, Bewley (1999) document's *symmetric* wage inflexibility. Second, *both management of firms as well as employed workers resist wage adjustments in response to unemployment* while supporters of wage adjustment were the unions who sought to avoid lay-offs. Third, management's view was that wage flexibility is undesirable since it has long term implications to the firm's performance whereas laying off undesired workers is a short run problem that "walks out the door."

The rest of this paper contains two main results that are demonstrated for two economies with a flexible wage market: a purely competitive economy and a NKM with a monopolistic competitive market structure:

(i) First, all risk averse workers accept the firm's offer and commit to work at $t+1$.

(ii) Second, *and this is the crucial novelty*, I later show that expected wage offers *increase* expected profits of the firm hence, all risk neutral firms make such job offers. If a firm's objective incorporates the risk aversion of its stock holders, then a mutual hedging against risky wages is even more beneficial since a firm gains from higher expected profits and from increased utility due to reduced risk. In some sense, the

firm is a bigger beneficiary of the institution. Finally, I explain that the resulting equilibrium is perfect. This second conclusion is qualified in one way. The result depends upon the nature of shocks and holds for all shocks that impact production, demand and profits. There are shocks that affect equilibrium wage without a direct effect on profits and for these the result above is not true. This matter is explained later.

In short, the job offers stipulated above and their acceptance *constitute a perfect equilibrium*. With such equilibrium behavior adopted by all firms, the flexible wage auction market either disappears completely or is confined to “irregular” jobs, and most wages fluctuate only in response to limited factors. In the limit, when all flexible regular wage markets are closed, the option of laid off workers to take jobs in the flexible wage market is altered drastically. In a modern economy it is transformed either into normal job search among firms that compete for workers with multi period expected wage offers, *not with current one period wage offers*, or to accepting available “irregular” jobs and the risk of layoff becomes a major risk of involuntary unemployment. Closing of the flexible wage market also alters the perfection of the resulting equilibrium.

2.2 Optimal Behavior of Workers

If at t workers can avoid a risky flexible wage \mathbf{W}_{t+1} and work at $t+1$ for its expected value $E_t[\mathbf{W}_{t+1}|\mathbf{I}_t]$, then all risk averse workers accept such offer. This is the “Implicit Contract” result and the only question is one of perfection: when $t+1$ arrives, would workers stay with the firm if a higher wage is realized in the flexible wage market? A worker’s decision not to deviate is deduced from weighing several factors that change with the wage offered, with an offer’s duration and with the equilibrium flexible wage at that time:

- (i) $\mathbf{V}_t^{\mathbf{W}}$ - present value of the offer’s constant wage for the offer’s duration is a function of the offered wage;
- (ii) $\mathbf{V}_t^{\mathbf{I}}$ - present value of the offer’s insurance value: it declines with time;
- (iii) $\mathbf{V}_t^{\mathbf{O}}$ - present value of the offer’s option value: declines with duration but rises with time;
- (iv) $\mathbf{V}_t^{\mathbf{F}}$ - present value of permanent work in the flexible wage market is a function of current wage.

The option value is a worker’s valuation of the *priority to receive future wage offers from the expected wage firm*. This option is lost if a worker deviates from the equilibrium. The condition $\mathbf{V}_t^{\mathbf{W}} + \mathbf{V}_t^{\mathbf{I}} + \mathbf{V}_t^{\mathbf{O}} > \mathbf{V}_t^{\mathbf{W}}$ is needed for offer’s acceptance but $\mathbf{V}_t^{\mathbf{W}} + \mathbf{V}_t^{\mathbf{I}} + \mathbf{V}_t^{\mathbf{O}} > \mathbf{V}_t^{\mathbf{W}}$ at all t is a condition of perfection with probability 1. This is restrictive since low probability events may reverse the inequality at some t . To relax it one can define perfection either with confidence limit or with a finite but very long exclusion period T^* after which a deviation is forgiven.

I note that after the offer date $\mathbf{V}_t^{\mathbf{I}}$ declines to zero, reached at the offer’s expiration and then it rises again when the offer is renewed. A longer offer’s duration increases $\mathbf{V}_t^{\mathbf{I}}$ but decreases $\mathbf{V}_t^{\mathbf{O}}$ since the option value rises

with frequency of offer revisions. For duration of one period the *option value always dominates*. If the flexible wage turns out higher than the offered wage a worker already has a revised offer for next period, which he prefers over a risky wage. Hence the option value is sufficient to ensure perfection. As offer duration rises the option value decreases but since shocks are Markov, they have geometric mean reversion property hence the flexible wage returns to steady state at which time the worker regrets a deviation from equilibrium. I will later show that offers of infinite durations cannot be a perfect equilibrium. Technical conditions to ensure perfection at optimal finite durations are not essential since the evidence shows most wage offers are for one year. This social norm is affected by other factors that promote perfection: moving cost and contribution of potential seniority to the option value. These marginally reduce incentives to deviate from the equilibrium.

The combination of seniority, corporate responsibility and sticky wages foster a culture of loyalty within the firm and hence the literature that stresses the role of loyalty in explaining wage stickiness (e.g. Solow (1979), Akerloff (1982), Akerloff and Yellen (1988),(1990), Bewley (1999)) does not conflict with my views. This literature assumes that *higher wages increase productivity* hence the firm does not lower wages to avoid reduced productivity. This assumption is too strong, it is questionable and is actually not needed. The internal scale of wages by skill and experience within the firm is an important mechanism that enforces perfection since it introduces an additional moving cost for a worker who considers taking advantage of higher flexible wage.

The risk of layoff is, for now, neglected. When there is an alternative flexible wage market, being laid off entails negligible added cost relative to the market with flexible wages. This is so since a worker would face the same layoff risk and the same implied search cost had he accepted a flexible wage offer at the outset. That is, a worker's layoff cost are essentially the same in the flexible wage markets as when a single firm deviates with a strategy of expected wage offers.

Given the observed degree of risk aversion, small mobility cost and expected higher wage due to seniority, all ensure workers accept and, most likely, honor the expected wage job offer.

3. Optimal Behavior of Firms and Properties of the Expected Wage Equilibrium

Why would a firm make expected wage job offers? Standard textbooks show a firm's profit function is convex in prices hence it prefers gambles offered by random input prices over the certainty of buying inputs at mean prices since such gambles raise expected profits. Such argument also implies a rational risk neutral firm will never hedge input purchases since with a convex profit function it is unprofitable to pay the insurance premium which a risk averse market demands for insuring against price fluctuations. But then, how do we explain

the fact that airlines hedge their fuel position with forward or futures markets and the classical baker hedges by buying wheat on the future market? Also, how do we explain the fact that most firms hedge major input cost by using diverse insurance contracts such as direct insurance, futures contracts or other financial derivatives?

One explanation for the *widely* observed hedging behavior of firms is that firms have concave utility of profits and hence the argument presumes managerial risk aversion overcomes the convexity of profits in prices. Managerial risk aversion is a reasonable assumption. The striking fact, however, is that the observed hedging behavior of firms is also explained by the results of this paper that demonstrate that an expected wage setting increases expected profits and hedging labor cost is optimal even for a risk neutral firm if hedging cost are low. It means an added factor exists that alters the standard argument about convexity of profits. Expected wage offer is then a *mutual insurance* of a firm with its workers *that benefits both sides*, not only workers. Indeed, under managerial risk aversion the firm's gain is asymmetric with the gain of workers since a firm has a double gain: one from concavity of the utility of profits and a second from the increased expected profits due to the expected wage offer. The gain of workers is due only to the concavity of the utility of consumption.

To understand the above results keep in mind the standard argument. It says convexity of profit function implies that a firm reduces use of expensive inputs and increases quantity employed of cheaper inputs, in addition to altering its production level when input prices change. When an input price is low, optimizing firms use more of this input and increase output. When a price is high, firms optimize by avoiding that input and decreasing output. This reasoning ignores the equilibrium relations between shocks and input prices. Input prices fluctuate in response to economy-wide shocks hence, in assessing riskiness of wage fluctuations *a rational firm takes into account the general equilibrium effects of shocks* that induce correlation of prices with shocks. Such correlation is ignored in formal statements of the convexity of profit function since it is usually assumed markets are complete and all independent effects of the shocks are traded by the firm with contingent claims. It is then suggested that the convexity of profits is an incorrect basis for judging if it is optimal for a firm to hedge the effect of risky shocks on prices. The key technical result proved below is that once equilibrium effects of shocks on prices are taken into account then, for all shocks with effects on output such as technology and demand shocks, *profit functions are convex in prices but concave in shocks!* Hence, optimizing firms *benefit from hedging the effects of the shocks*. This is not sufficient to prove the firm will benefit from hedging price fluctuations of major inputs such as wages since control of wages provides only partial insurance and a separate argument is developed to prove that hedging wage risk is in fact optimal and making expected wage offers is an efficient way to accomplish it. Although there are shocks for which hedging is not optimal, I show that for all practical parameter

values the effect of such shocks is negligible.

An equilibrium with expected wage offers constitutes a new institutional arrangement with two crucial features: (i) the firm's high labor cost are hedged, and (ii) the firm is free to optimally select its employment and output levels allowing it to make quantity adjustments as fast as it needs. A firm's survival depend upon its ability to respond rapidly to drastic changes. If prices, public policy and the environment (i.e. legal, regulatory etc) adjust rapidly to shocks, the firm's risk is reduced. But since prices and the institutional environment adjust only slowly, the main tool left for the firm are adjustments in quantities.

Finally, how can the firm and its workers gain by deviating from a competitive flexible wage market? The answer is that the flexible wage market is not an Arrow-Debreu market since it lacks a complete set of contingent claims. The gains of the firm and workers result from their Pareto improving mutual insurance institution. This conclusion is modified when all firms deviate from the flexible wage market, leading this market to close and involuntary unemployment to emerges. When this occurs the real cost of the mutual insurance is born by the unemployed who become the problem of society at large.

3.1 A Simple Schematic Argument

The results above depend upon market structure and the nature of the shocks and there are hypothetical conditions when the proposition is false. I thus present first a simple schematic argument to explain the key idea. To that end consider any competitive economy in which a firm produces some outputs using labor and N other inputs. The indirect maximized real profit function is

$$\Pi(W_t, p_t^1, p_t^2, \dots, p_t^N, \zeta_t).$$

Nominal values are ignored for now. Real wage W_t and prices p_t^j are relative to price level, and $\zeta_t = (\zeta_t^1, \zeta_t^2, \dots, \zeta_t^H)$ are mean 1 random shocks. Although $\Pi(W_t, p_t^1, p_t^2, \dots, p_t^N, \zeta_t)$ is a convex function of prices, expected profits depend upon macroeconomic factors that determine the real wages and relative prices. Hence, to assess expected profits a firm considers the equilibrium structure of the economy within which wages and prices are determined, and this includes monetary and fiscal policy. At this stage I add only the standard equilibrium condition requiring excess demand to equal zero hence specifying (N+1) equations $\mathcal{E}(W_t, p_t^1, p_t^2, \dots, p_t^N, \zeta_t) = \mathbf{0}$ from which to deduce the equilibrium map:

$$(1) \quad (W_t, p_t^1, \dots, p_t^N) = \Psi(\zeta_t^1, \zeta_t^2, \dots, \zeta_t^H).$$

One uses (1) to obtain a reduced form profit function of a flexible wage economy that is written in the form $\Pi(W_t, p_t^1, p_t^2, \dots, p_t^N, \zeta_t) = \hat{\Pi}(\zeta_t)$. It shows all profit risks are actually due to the shocks ζ_t and the key technical

result explained later is that *although* $\Pi(W_t, p_t^1, p_t^2, \dots, p_t^N, \zeta_t)$ is convex in $(W_t, p_t^1, p_t^2, \dots, p_t^N)$, *the reduced form profit function* $\hat{\Pi}(\zeta_t)$ *is concave in the shocks* ζ_t . This result is important but not sufficient since the firm can control only the wage it offers, not the shock. This means that in making wage offers that differ from the above competitive flexible wage rates a firm alters the impact of convexity with respect to prices. I then write the profit function in a form that *keeps the wage as a separate variable*, while recognizing that for competitive firms the wage is a function of the state variables only:

$$(2) \quad \Pi(W_t, p^1(\zeta_t), p^2(\zeta_t), \dots, p^N(\zeta_t), \zeta_t) \quad , \quad W_t = W(\zeta_t) \text{ for a competitive firm.}$$

Using (2) I can now study the effect of a firm's deviation from the competitive wage by offering its workers a separate wage function. Specifically, assume all agents hold the same probability belief then, instead of a risky flexible wage $W_{t+1} = W(\zeta_{t+1})$ at $t+1$, the firm offers its workers at t to work at $t+1$ for a *certain* wage that equals the conditionally expected competitive wage denoted $W_t^e \equiv E_t[W(\zeta_{t+1})|I_t]$ that depends upon information at t . Expected wage offers for two or more periods are formulated later. To see the impact of the firm's offer for it computes the difference in expected profits under the two environments:

$$(3a) \quad \Delta_t(1) = E_t \left[\Pi(W(\zeta_{t+1}), p^1(\zeta_{t+1}), p^2(\zeta_{t+1}), \dots, p^N(\zeta_{t+1}), \zeta_{t+1} | I_t) - \Pi(W_t^e, p^1(\zeta_{t+1}), p^2(\zeta_{t+1}), \dots, p^N(\zeta_{t+1}), \zeta_{t+1} | I_t) \right].$$

For risk assessment a second order approximation around steady state is appropriate. If we expand the two profit function up to second order, then take the conditional expected value of the difference, all first order terms and all terms that do not involve the wage rate are cancelled. The difference contains only two terms

$$(3b) \quad \Delta_t(1) = \frac{1}{2} \bar{\Pi}_{ww} \left[E_t[(W_{t+1} - \bar{W})^2 | I_t] - (W_t^e - \bar{W})^2 \right] + \sum_{j=1}^H \bar{\Pi}_{w\zeta^j} \left[E_t[(W_{t+1} - \bar{W})(\zeta_{t+1}^j - 1) | I_t] - (W_t^e - \bar{W}) E_t[(\zeta_{t+1}^j - 1) | I_t] \right].$$

The first is a standard positive effect of convex profit functions. It takes into account the difference in volatility between the flexible wage and the expected wage offer. The second is the one I focus on here: it considers the economy wide factors that impact the competitive wage rate and the correlation of the competitive wage with the shocks. *These are the factors that are altered by an expected wage offer and if they have an impact on expected profits, this impact come in addition to the effect of convexity.*

To explain (3b), note that when a firm considers effect of shocks on production – central to this work – changed productivity or demand and changed wages are viewed as different events and convexity of profits with respect to wages is defined given all other variables (including ζ_t) held fixed. To profit from wage fluctuations, a firm *reduces output* when wages are high and *increases output* when wages are low. But when it considers all the effect of shocks, a firm finds that time of high wage is just the time when productivity is high and then it wants to *increase output*, whereas the time of low wage is just the time when productivity is low and the firm wants to

reduce output. This is when ζ in (3b) are proxies for change in output and the problem is due to the positive correlation between output and wages.

It is clear the effects of wage fluctuations on profits depend upon the economic environment and hence on the nature of the shocks, the market structure in which the firm operates, public policy in general and interest rates and monetary policy in particular. There are shocks that imply $\bar{\Pi}_{\mathbf{w}^j} = \mathbf{0}$ and hedging against such risks is not optimal. It will be proved that *relative to major shocks that constitute realistic economic risks* $\Delta_t(\mathbf{1}) < \mathbf{0}$ in (3b) and as long as such shocks are present, it is optimal for the firm to hedge its labor cost.

3.2 A Competitive Model with Production Shocks: One Period Ahead Offers

Since conditions that ensure $\Delta_t(\mathbf{1}) < \mathbf{0}$ in (1.3b) depend upon the economic environment, I consider first the simplest setting of a competitive economy with which to explain how the main argument works. Assume that there is a large number n of identical firms with production function

$$(4a) \quad Y_t = \zeta_t N_t^{1-\alpha}$$

$$(4b) \quad \zeta_{t+1} - 1 = \lambda_\zeta (\zeta_t - 1) + \rho_{t+1}^\zeta, \quad \rho_{t+1}^\zeta \text{ i.i.d, mean 0 and variance } \sigma_\zeta^2.$$

A familiar way of stating (4b) is to assume either that $\hat{\zeta}_t \equiv \log(\zeta_t) \equiv (\zeta_t - 1)$ is a Markov process or to require that ζ_t is conditionally log-normal. There is also a large number m of identical households with utility function

$$(4c) \quad \left(\frac{1}{1-\sigma} (C_t)^{1-\sigma} - \frac{1}{1+\eta} (L_t)^{1+\eta} \right).$$

Assume the economy has one exogenous production shock and think of it as a vector of shocks that induce output and profit risks. These include technology shocks, demand shocks but also shocks to production support services such as financing. There is no markets for contingent claims and no trading in assets. In equilibrium all output is consumed and aggregate labor supplied equals aggregate labor demand. Hence, for all i and j I require

$$C_{it} = \frac{n}{m} Y_{jt}, \quad L_{it} = \frac{n}{m} N_{jt}.$$

The main results are shown in Step 1 below to be independent of $\mathbf{k} = \frac{n}{m}$ and hence, without loss of generality, I set $\mathbf{k} = 1$ in the rest of the argument below. In the development below I also ignore all indices of i and j since all agents are identical. The computations are carried out in three steps.

Proposition 1: (i) $\bar{\Pi}_{\mathbf{w}^j} > \mathbf{0}$ but a flexible wage competitive profit function $\mathbf{W}(\zeta_t)$ is concave in ζ_t ;
(ii) If $\eta + \sigma + 2\alpha(1-\sigma) > 0$ then $\Delta_t(\mathbf{1}) < \mathbf{0}$ in (3b), this expression is proportional to σ_ζ^2 and is independent of t . It is then optimal for a firm to make expected wage offers and depart from the flexible wage market.

The proof is developed in two steps.

Step 1: Deriving profit functions and equilibrium conditions of the competitive market

The real profit function is

$$(5a) \quad \Pi_t = \zeta_t N_t^{1-\alpha} - W_t N_t$$

and all have the same fixed factor, allowing positive profits. Optimal labor supply requires

$$(6a) \quad (C_t)^{-\sigma} W_t = (L_t)^\eta$$

while optimal labor demand requires that

$$(6b) \quad (1-\alpha)\zeta_t N_t^{-\alpha} = W_t \quad \Rightarrow \quad N_t(W, \zeta_t) = (1-\alpha)^{\frac{1}{\alpha}} \zeta_t^{\frac{1}{\alpha}} (W_t)^{-\frac{1}{\alpha}}.$$

For the moment, ignore (1.6a). Inserting (1.6b) into (1.5a) and taking the wage rate as given, define the *maximized* profit function where optimal employment and output are defined in terms of the wage rate

$$(5b) \quad \begin{aligned} \Pi_t(W_t, \zeta_t) &= \zeta_t [(1-\alpha)^{\frac{1}{\alpha}} \zeta_t^{\frac{1}{\alpha}} (W_t)^{-\frac{1}{\alpha}}]^{1-\alpha} - W_t [(1-\alpha)^{\frac{1}{\alpha}} \zeta_t^{\frac{1}{\alpha}} (W_t)^{-\frac{1}{\alpha}}] \\ &= \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} \zeta_t^{\frac{1-\alpha}{\alpha}} (W_t)^{-\frac{1-\alpha}{\alpha}}. \end{aligned}$$

The distinction between the wage and the shock in (5b) is used in the argument below since *a firm that deviates from the competitive labor market and sets its own wage function controls the wage in (5b) but not the shock.*

Observe that since by (5a) $\Pi_{W_t} = -N_t$, (5b) exhibits the convexity of profits with respect to the real wage

$$\Pi_{W_t W_t} = \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} \zeta_t^{\frac{1}{\alpha}} (W_t)^{-\frac{1}{\alpha}} > 0.$$

I now take the key step and introduce market clearing conditions $C = kY$, $L = kN$. (5a)-(5b) imply

$$W_t = (N_t)^\eta (Y_t)^\sigma k^{\eta+\sigma}, \quad (1-\alpha)\zeta_t \left(\frac{Y_t}{\zeta_t}\right)^{-\frac{\alpha}{1-\alpha}} = (Y_t)^\sigma \left(\frac{Y_t}{\zeta_t}\right)^{\frac{\eta}{1-\alpha}} k^{\eta+\sigma}.$$

I then deduce the solution of output in a flexible wage market

$$(7a) \quad Y_t = \left(\frac{1-\alpha}{k}\right)^{\frac{1-\alpha}{\eta+\alpha+(1-\alpha)\sigma}} \zeta_t^{\frac{1+\eta}{\eta+\alpha+(1-\alpha)\sigma}}, \quad \bar{Y} = \left(\frac{1-\alpha}{k}\right)^{\frac{1-\alpha}{\eta+\alpha+(1-\alpha)\sigma}}.$$

Let $k_W = (k)^{\frac{\alpha}{\eta+\alpha+(1-\alpha)\sigma}}$ and the solution for the competitive flexible wage rate is

$$(7b) \quad W_t = k_W (1-\alpha)^{\frac{\eta+(1-\alpha)\sigma}{\eta+\alpha+(1-\alpha)\sigma}} \zeta_t^{\frac{\eta+\sigma}{\eta+\alpha+(1-\alpha)\sigma}}, \quad \bar{W} = k_W (1-\alpha)^{\frac{\eta+(1-\alpha)\sigma}{\eta+\alpha+(1-\alpha)\sigma}}.$$

Now insert (7b) into the profit function to deduce

$$\Pi_t(W_t, \zeta_t) = \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} \zeta_t^{\frac{1-\alpha}{\alpha}} \left[k_W (1-\alpha)^{\frac{\eta+(1-\alpha)\sigma}{\eta+\alpha+(1-\alpha)\sigma}} \zeta_t^{\frac{\eta+\sigma}{\eta+\alpha+(1-\alpha)\sigma}} \right]^{\frac{1-\alpha}{\alpha}}.$$

As anticipated, this reduced form profit function is *concave* in ζ_t .

It follows from (4b) that the distributions of output and wage deviations from steady state are

$$(8a) \quad (Y_t - \bar{Y}) = \bar{Y}A(\zeta_t - 1) \quad , \quad A = \frac{1 + \eta}{\eta + \alpha + (1 - \alpha)\sigma} > 0$$

$$(8b) \quad (W_t - \bar{W}) = \bar{W}B(\zeta_t - 1) \quad , \quad B = \frac{\eta + \sigma}{\eta + \alpha + (1 - \alpha)\sigma} > 0.$$

hence (8b) implies that

$$(8c) \quad E_t(W_{t+1} - \bar{W}) = \bar{W}B[E_t(\zeta_{t+1} - 1)] = \lambda_\zeta \bar{W}B(\zeta_t - 1) = \lambda_\zeta(W_t - \bar{W})$$

which is an important relation used below. It follows from (4a)-(4b) that (8b) imply that in the flexible wage economy the statistics of conditional expectations would show the following results:

$$E_t(W_{t+1} - \bar{W})(\zeta_{t+1} - 1) = \bar{W}B[\lambda_\zeta^2(\zeta_t - 1)^2 + \sigma_\zeta^2] \quad , \quad E_t(W_{t+1} - \bar{W})^2 = \bar{W}^2B^2[\lambda_\zeta^2(\zeta_t - 1)^2 + \sigma_\zeta^2]$$

These are deduced for one Markov state variable. A generalization to multiple state variables is immediate.

I next use a *second* order approximation of optimal profit function but assumption (4b) about the distribution of $\log(\zeta_t)$ imply *linearity* of (8a)-(8b). For more general distributions, (8a)-(8b) would be *first order* approximations. The Markov assumption (4b), is entirely standard.

Step 2: Computing the gain of deviation from the flexible wage market

I first state the exact mathematical problem (3a). The firm computes the difference in expected profits

$$\Pi_{t+1}^F(W_{t+1}, \zeta_{t+1}) - \Pi_{t+1}^{EW}(E_t(W_{t+1}), \zeta_{t+1}) = \alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} \zeta_{t+1}^\alpha (W_{t+1})^{-\frac{1-\alpha}{\alpha}} - \alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} \zeta_{t+1}^\alpha (E_t(W_{t+1}))^{-\frac{1-\alpha}{\alpha}}.$$

A closed form solution cannot be obtained and a second order approximation is the only way to proceed. Hence, expand the profit function up to second order around the steady state values $(\bar{\Pi}, \bar{W}, \bar{\zeta})$. For the flexible wage firm the profit function is approximated by

$$(9a) \quad \Pi_{t+1}^F - \bar{\Pi} = \Pi_W(W_{t+1} - \bar{W}) + \Pi_\zeta(\zeta_{t+1} - \bar{\zeta}) + \frac{1}{2}\Pi_{WW}[(W_{t+1} - \bar{W})^2] + \\ + \Pi_{W\zeta}[(W_{t+1} - \bar{W})(\zeta_{t+1} - \bar{\zeta})] + \frac{1}{2}\Pi_{\zeta\zeta}(\zeta_{t+1} - \bar{\zeta})^2$$

whereas the profits of the expected wage firm are approximated by

$$(9b) \quad \Pi_{t+1}^{EW} - \bar{\Pi} = \Pi_W(W_t^e - \bar{W}) + \Pi_\zeta(\zeta_{t+1} - \bar{\zeta}) + \frac{1}{2}\Pi_{WW}[(W_t^e - \bar{W})^2] + \\ + \Pi_{W\zeta}[(W_t^e - \bar{W})(\zeta_{t+1} - \bar{\zeta})] + \frac{1}{2}\Pi_{\zeta\zeta}(\zeta_{t+1} - \bar{\zeta})^2.$$

The aim is to assess $\Delta_t(1) = E_t[\Pi_{t+1}^F - \bar{\Pi} | I_t] - E_t[\Pi_{t+1}^{EW} - \bar{\Pi} | I_t, W_{t+1} = W_t^e]$. The condition $W_{t+1} = W_t^e$ states the firm's real wage at t+1 is fixed in advance at the conditionally expected wage at date t. Compute the difference, evaluate at steady state values and rearrange to find that

$$(10) \quad \Delta_t(1) = (1/2)\Pi_{\mathbf{w}\mathbf{w}}E_t[(\mathbf{W}_{t+1}-\bar{\mathbf{W}})^2 - (\mathbf{W}_t^e - \bar{\mathbf{W}})^2] + \Pi_{\mathbf{w}\zeta}E_t[(\mathbf{W}_{t+1}-\bar{\mathbf{W}})(\zeta_{t+1}-1) - (\mathbf{W}_t^e - \bar{\mathbf{W}})(\zeta_{t+1}-1)].$$

Now use (1.5b) to have

$$(11a) \quad (\Pi_{\mathbf{w}\mathbf{w}})_t = \frac{1}{\alpha}(1-\alpha)^{\frac{1}{\alpha}}\zeta_t^{\frac{1}{\alpha}}(\mathbf{W}_t)^{-\frac{1+\alpha}{\alpha}} \quad \text{hence in steady state } \Pi_{\mathbf{w}\mathbf{w}} = \frac{1}{\alpha}(1-\alpha)^{\frac{1}{\alpha}}(\bar{\mathbf{W}})^{-\frac{1+\alpha}{\alpha}} > 0$$

$$(11b) \quad (\Pi_{\mathbf{w}\zeta})_t = -\frac{1}{\alpha}(1-\alpha)^{\frac{1}{\alpha}}\zeta_t^{\frac{1-\alpha}{\alpha}}(\mathbf{W}_t)^{-\frac{1}{\alpha}} \quad \text{hence in steady state } \Pi_{\mathbf{w}\zeta} = -\frac{1}{\alpha}(1-\alpha)^{\frac{1}{\alpha}}(\bar{\mathbf{W}})^{-\frac{1}{\alpha}} < 0.$$

Now evaluate the difference in (10) using (7b)-(8b) for all $\alpha < \frac{1}{2}$, $\eta > 0$, $\sigma > 0$ as follows:

$$\begin{aligned} \Delta_t(1) &= \frac{1}{2} \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{\mathbf{W}})^{-\frac{1+\alpha}{\alpha}} E_t[(\mathbf{W}_{t+1}-\bar{\mathbf{W}})^2 - (\mathbf{W}_t^e - \bar{\mathbf{W}})^2] \\ &\quad - \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{\mathbf{W}})^{-\frac{1}{\alpha}} E_t[(\mathbf{W}_{t+1}-\bar{\mathbf{W}})(\zeta_{t+1}-1) - (\mathbf{W}_t^e - \bar{\mathbf{W}})(\zeta_{t+1}-1)] \end{aligned}$$

Now

$$E_t[(\mathbf{W}_{t+1}-\bar{\mathbf{W}})^2 - (\mathbf{W}_t^e - \bar{\mathbf{W}})^2] = \bar{\mathbf{W}}^2 \mathbf{B}^2 E_t[(\zeta_{t+1}-1)^2] - \bar{\mathbf{W}}^2 \mathbf{B}^2 \lambda_\zeta^2 (\zeta_t - 1)^2 = \bar{\mathbf{W}}^2 \mathbf{B}^2 \sigma_\zeta^2$$

$$E_t[(\mathbf{W}_{t+1}-\bar{\mathbf{W}})(\zeta_{t+1}-1) - (\mathbf{W}_t^e - \bar{\mathbf{W}})(\zeta_{t+1}-1)] = \bar{\mathbf{W}} \mathbf{B} E_t[(\zeta_{t+1}-1)^2] - \bar{\mathbf{W}} \mathbf{B} (E_t \zeta_{t+1} - 1)^2 = \bar{\mathbf{W}} \mathbf{B} [\lambda_\zeta^2 (\zeta_t - 1)^2 + \sigma_\zeta^2 - \lambda_\zeta^2 (\zeta_t - 1)^2] = \bar{\mathbf{W}} \mathbf{B} \sigma_\zeta^2.$$

Combine the two to conclude

$$\begin{aligned} \Delta_t(1) &= \frac{1}{2} \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{\mathbf{W}})^{-\frac{1+\alpha}{\alpha}} \bar{\mathbf{W}}^2 \mathbf{B}^2 \sigma_\zeta^2 - \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{\mathbf{W}})^{-\frac{1}{\alpha}} \bar{\mathbf{W}} \mathbf{B} \sigma_\zeta^2 \\ &= \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{\mathbf{W}})^{-\frac{1-\alpha}{\alpha}} \mathbf{B} (\sigma_\zeta^2) \left[\frac{1}{2} \mathbf{B} - 1 \right] \\ &= \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{\mathbf{W}})^{-\frac{1-\alpha}{\alpha}} \mathbf{B} (\sigma_\zeta^2) \left[\frac{1}{2} \left(\frac{\eta + \sigma}{(\eta + \alpha) + (1-\alpha)\sigma} \right) - 1 \right] < 0. \end{aligned}$$

The conclusion follows from $\frac{1}{2} \mathbf{B} - 1 < 0$, a relation that will appear again. I have thus shown a firm prefers an expected wage, fixed in advance for one period, and *the gain is independent of the current wage*. The conditions $\eta > 0$, $\sigma > 0$ are obvious and $\alpha < \frac{1}{2}$ is natural but far too strong. All that is needed for the result above is $(\eta + \sigma) + 2\alpha(1 - \sigma) > 0$, a condition that holds for any relevant parameter configurations.

3.3 Extending the Argument to Longer Offer Durations

For clarity I carry out the computation for two periods before showing how to generalize to N periods. At date t the expected present value of two period profits of a competitive flexible wage firm is

$$\beta E_t[\Pi_{t+1}^F + \beta \Pi_{t+2}^F | I_t].$$

The expected wage firm then considers deviating by offering a constant wage for two periods. To compute the offered future wage starting at $t+1$ use the computed solution of the flexible wage in (18c) to deduce:

$$W_t^e(2) - \bar{W} = \frac{1}{1+\beta} E_t[(W_{t+1} - \bar{W}) + \beta(W_{t+2} - \bar{W})] = \left(\frac{1+\beta\lambda_\zeta}{1+\beta}\right) \lambda_\zeta (W_t - \bar{W}).$$

I divide by $(1+\beta)$ since actual wage is paid at t and in the future, at $t+1$. The analogue of (10) for two periods (with a notation $\Delta_t(2)$ for present value of the difference) is defined by

$$(12) \quad \Delta_t(2) = (1/2) \Pi_{\bar{W}\bar{W}} E_t[(W_{t+1} - \bar{W})^2 + \beta(W_{t+2} - \bar{W})^2 - [(W_t^e(2) - \bar{W})^2 + \beta(W_t^e(2) - \bar{W})^2]] \\ + \Pi_{\bar{W}\zeta} E_t[(W_{t+1} - \bar{W})(\zeta_{t+1} - 1) + \beta(W_{t+2} - \bar{W})(\zeta_{t+2} - 1)] - [(W_t^e(2) - \bar{W})(\zeta_{t+1} - 1) + \beta(W_t^e(2) - \bar{W})(\zeta_{t+2} - 1)]].$$

By rearranging the terms in (12) I evaluate for $k=1, 2$ the following two general terms

$$(13a) \quad \frac{1}{2} \Pi_{\bar{W}\bar{W}} E_t[(W_{t+k} - \bar{W})^2 - (W_t^e(2) - \bar{W})^2] = \frac{1}{2} \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1+\alpha}{\alpha}} \bar{W}^2 B^2 [\lambda_\zeta^{2k} (\zeta_t - 1)^2 + \sigma_\zeta^2 \sum_{j=0}^{k-1} \lambda_\zeta^{2j} - \left(\frac{1+\beta\lambda_\zeta}{1+\beta}\right)^2 \lambda_\zeta^{2k} (\zeta_t - 1)^2]$$

$$(13b) \quad \Pi_{\bar{W}\zeta} [E_t(W_{t+k} - \bar{W})(\zeta_{t+k} - 1) - E_t(W_t^e(2) - \bar{W})(\zeta_{t+k} - 1)] = -\frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1+\alpha}{\alpha}} \bar{W} B \left[\lambda_\zeta^{2k} (\zeta_t - 1)^2 + \sigma_\zeta^2 \sum_{j=0}^{k-1} \lambda_\zeta^{2j} \right] \\ + \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1+\alpha}{\alpha}} \bar{W} B \frac{1+\beta\lambda_\zeta}{1+\beta} \lambda_\zeta^{k+1} (\zeta_t - 1)^2$$

Sum over k , multiply the second term by β , to have

$$\Delta_t(2) = \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1+\alpha}{\alpha}} B \left[(1+\beta\lambda_\zeta^2) \lambda_\zeta^2 (\zeta_t - 1)^2 + \sigma_\zeta^2 (1+\beta\lambda_\zeta^2) \right] \left(\frac{1}{2} B - 1\right) \\ - \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1+\alpha}{\alpha}} B \left[\frac{(1+\beta\lambda_\zeta)^2}{(1+\beta)^2} \lambda_\zeta^2 (\zeta_t - 1)^2 \right] \left(\frac{1}{2} B - 1\right).$$

Hence, the gain is

$$(14) \quad \Delta_t(2) = \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1+\alpha}{\alpha}} B \left[\left(\frac{\beta(1-\lambda_\zeta)^2}{1+\beta}\right) \lambda_\zeta^2 (\zeta_t - 1)^2 + \sigma_\zeta^2 (1+\beta\lambda_\zeta^2) \right] \left(\frac{1}{2} B - 1\right) < 0$$

The term $\frac{1}{2} B - 1 < 0$ continues to play a key role. However, the gain in expected profits is now different from the case of one period in varying over time as it depends upon ζ_t or, equivalently, on the current wage. More general, *multi period wage offers depend upon the starting position* and this is not surprising: if workers and the firm commit to a long term wage, it will take into account the dynamics of market wages,

Would the firm make an infinite horizon offer? Any expected wage offer cannot have first order effects since then it may either not be optimal for a firm to make it or for workers to accept it. Hence it must satisfy

$$(15) \quad \sum_{k=1}^{\infty} \beta^k (W_t^e(\infty) - \bar{W}) = \sum_{k=1}^{\infty} \beta^k E_t(W_{t+k} - \bar{W}) \Rightarrow (W_t^e(\infty) - \bar{W}) = \frac{1-\beta}{\beta} \sum_{k=1}^{\infty} \beta^k E_t(W_{t+k} - \bar{W})$$

and by (8c)

$$E_t(W_{t+k} - \bar{W}) = \lambda_{\zeta}^k (W_t - \bar{W}).$$

It follows that date t expected flexible wage at future date $(t+k)$ converges in this model to steady state wage or to *trend productivity* in an equivalent growing economy. It is next seen that an infinite horizon offer remains bounded away from steady state at all future dates since

$$(16) \quad (W_t^e(\infty) - \bar{W}) = (1-\beta) \sum_{k=1}^{\infty} \beta^{k-1} E_t(W_{t+k} - \bar{W}) = \frac{1-\beta}{\beta} \sum_{k=1}^{\infty} \beta^k \lambda_{\zeta}^k (W_t - \bar{W}) = \frac{1-\beta}{1-\beta\lambda_{\zeta}} \lambda_{\zeta} (W_t - \bar{W}) = \frac{1-\beta}{1-\beta\lambda_{\zeta}} \lambda_{\zeta} \bar{W} B (\zeta_t - 1).$$

For parameter values $\beta = 0.99$, $\lambda_{\zeta} = 0.90$ an infinite horizon offer is a constant wage with a deviation from steady state that is about 16% of the deviation of the flexible wage from steady state at the date of the offer. This is a significant distance from current wage. The fact that $W_t^e(\infty) \neq \bar{W}$ hence bounded away from steady state, is due to the fact the firm and the workers place heavier weight on near term movement of the competitive flexible wage in our imaginary auction market, but it will raise significant problems of perfection to be discussed shortly.

To compute the gain from of an infinite horizon offer, note that a direct extension of (12) imply that

$$\Delta_t(\infty) = (1/2) \Pi_{\bar{W}} \sum_{k=1}^{\infty} \beta^{k-1} E_t[(W_{t+k} - \bar{W})^2 - (W_t^e(\infty) - \bar{W})^2] + \Pi_{\bar{W}\zeta} \sum_{k=1}^{\infty} \beta^{k-1} E_t[(W_{t+k} - \bar{W})(\zeta_{t+k} - 1) - (W_t^e(\infty) - \bar{W})(\zeta_{t+k} - 1)]$$

The infinite sum leads to some simplifications. The k 'th term of the sum has the form

$$\begin{aligned} \frac{1}{2} \Pi_{\bar{W}} E_t[(W_{t+k} - \bar{W})^2 - (W_t^e(\infty) - \bar{W})^2] &= \frac{1}{2} \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1+\alpha}{\alpha}} [\lambda_{\zeta}^{2k} (W_t - \bar{W})^2 + \bar{W}^2 B^2 \sigma_{\zeta}^2 \sum_{j=0}^{k-1} (\lambda_{\zeta}^{2j}) - (\frac{1-\beta}{1-\beta\lambda_{\zeta}} \lambda_{\zeta} (W_t - \bar{W}))^2] \\ \Pi_{\bar{W}\zeta} [E_t(W_{t+k} - \bar{W})(\zeta_{t+k} - 1) - E_t(W_t^e(\infty) - \bar{W})(\zeta_{t+k} - 1)] &= -\frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} \bar{W}^{-\frac{1}{\alpha}} \left[\lambda_{\zeta}^{2k} \frac{(W_t - \bar{W})^2}{\bar{W} B} + \sigma_{\zeta}^2 \sum_{j=0}^{(k-1)} \lambda_{\zeta}^{2j} \right] \\ &\quad + \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} \bar{W}^{-\frac{1}{\alpha}} \frac{1-\beta}{1-\beta\lambda_{\zeta}} \lambda_{\zeta}^{k+1} \frac{(W_t - \bar{W})^2}{\bar{W} B}. \end{aligned}$$

Now use (7b)-(8b), multiply each term by β^{k-1} and sum over k

$$\begin{aligned} \Delta_t(\infty) &= \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1-\alpha}{\alpha}} B \left[\frac{\lambda_{\zeta}^2}{1-\beta\lambda_{\zeta}^2} (\zeta_t - 1)^2 + \sigma_{\zeta}^2 \sum_{k=1}^{\infty} \sum_{j=0}^{(k-1)} \lambda_{\zeta}^{2j} \right] \left(\frac{1}{2} B - 1 \right) \\ &\quad - \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1-\alpha}{\alpha}} B \left[\frac{1-\beta}{(1-\beta\lambda_{\zeta})^2} \lambda_{\zeta}^2 (\zeta_t - 1)^2 \right] \left(\frac{1}{2} B - 1 \right) \end{aligned}$$

hence

$$(17) \quad \Delta_t(\infty) = \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1-\alpha}{\alpha}} B \left[\frac{2\beta(1-\lambda_\zeta)\lambda_\zeta^2}{(1-\beta\lambda_\zeta^2)(1-\beta\lambda_\zeta)^2} (\zeta_t - 1)^2 + \sigma_\zeta^2 \sum_{k=1}^{\infty} \sum_{j=0}^{(k-1)} \lambda_\zeta^{2j} \right] \left(\frac{1}{2} B - 1 \right) < 0.$$

I then sum up the results as follows:

Proposition 2: Given a competitive economy with a flexible wage market, if $\eta + \sigma + 2\alpha(1-\sigma) > 0$ then $\Delta_t(T) < 0$ for all horizons T with wage offers that depend upon ζ_t . Hence for any horizon, it is optimal for a firm to offer and for workers to accept a weighted average expected wage.

Averaging with discounting as in (15) or using arithmetic average are virtually the same for a one year span. To see why consider realistic parameter values of $\beta = 0.99$, $\lambda_\zeta = 0.90$ for a quarterly model. Then

$$\begin{aligned} W_t^e(4) - \bar{W} &= \frac{1}{4} \sum_{k=1}^4 E_t[W_{t+k} - \bar{W}] = \frac{1}{4} \sum_{k=1}^4 \lambda_\zeta^k [W_t - \bar{W}] = (0.7738) [W_t - \bar{W}] \\ W_t^e(4) - \bar{W} &= \sum_{k=1}^4 \frac{\beta^{k-1}}{1+\beta+\beta^2+\beta^3} E_t[W_{t+k} - \bar{W}] = \sum_{k=1}^4 \frac{\beta^{k-1}\lambda_\zeta^k}{1+\beta+\beta^2+\beta^3} [W_t - \bar{W}] = (0.7747) [W_t - \bar{W}]. \end{aligned}$$

Even for two years the difference is very small.

3.4 Optimal Duration and Perfection

An infinite horizon offer is beneficial to both sides but will not be sustained as a perfect equilibrium. Result (8b) shows a flexible wage fluctuates around steady state (or trend productivity) while an infinite horizon offer (16), which is a *constant* present value of future expected wages, puts a heavy weight on current wage and remains *permanently* bounded away from steady state. Hence, with probability one a sequence of shocks will occur when current flexible wage is so far above or below the infinite horizon offer that either the firm or the workers have a strong enough motive to abandon the constant wage equilibrium. In either case the firm could not perform in accord with the required equilibrium.

It is then clear that a perfect memory implies an optimal wage offer are part of the conditions for equilibrium perfection under which successive expected wage offers stay within a reasonable distances of the flexible wages. This is done in two ways. First, an offer's duration cannot be too long and a one or two year duration is a reasonable convention. It gives the firm an opportunity to adjust any fixed wage in accord with a wage function defined by the firm's scale. Second, duration is altered if conditions change too rapidly and I noted the evidence (see Cecchetti (1984)) that the frequency of wage adjustments increases with inflation.

3.5 Shocks Without Incentive to Offer Expected Wage

The force of the theory developed here is that it applies to all *real economies*, not to all conceivable economies. It is then instructive to examine the effect of shocks that operate in the opposite direction and *reduce* a firm's incentive to hedge labor input. Two possible shocks I consider are to the discount rate and to marginal utility of consumption or labor. Although such shocks are used in the theoretical literature, no data sources exist that provide direct observations on them hence there are no reliable estimates how large or persistent they are. In all likelihood if such shocks ever occur they do not occur very often and they do they are not too large or persistent. A key property working against an expected wage offer is for the shock to *affect the wage and output in opposite directions*. That is, if a shock increases output but decreases the wage or, if the shock decrease output but increases the wage. In such a case the shocks conform to the impact of convexity which calls for using more of an input when it is cheap and less of it when it is expensive.

The first shock is then to the discount rate. Examination of the reasoning in the previous section shows that such shocks have no effect on the wage or on output since the model has no capital and no sticky prices that call for intertemporal choice. Random discounting impacts *perceived* risk of future flows but such perception cannot be hedged with a constant wage in our model. That is, such shocks have an effect when intertemporal choice is made but it has no effect in the model at hand.

The second shock is proportional to the marginal propensity to consume where the utility takes the form

$$\sum_{\tau=0}^{\infty} \beta^{\tau} \left(\frac{\Psi_{t+\tau} (C_{j,t+\tau})^{1-\sigma}}{1-\sigma} - \frac{1}{1+\eta} (L_{j,t+\tau})^{1+\eta} \right).$$

Optimal labor supply is

$$\Psi_t (C_t)^{-\sigma} W_t = (L_t)^{\eta}.$$

Assuming standard Markov transition

$$(\Psi_{t+1} - 1) = \lambda_{\Psi} (\Psi - 1) + \rho_{t+1}^{\Psi} \quad , \quad \rho_{t+1}^{\Psi} \text{ i.i.d, mean 0 and variance } \sigma_{\Psi}^2$$

the solution for output is deduced from

$$[(1-\alpha)\Psi_t] \zeta_t \left(\frac{Y_t}{\zeta_t} \right)^{-\frac{\alpha}{1-\alpha}} = (Y_t)^{\sigma} \left(\frac{Y_t}{\zeta_t} \right)^{\frac{\eta}{1-\alpha}}.$$

Hence, in a flexible wage market

$$(18a) \quad Y_t = (1-\alpha)^{A_0} \zeta_t^{A_{\zeta}} \Psi_t^{A_{\Psi}} \quad , \quad \bar{Y} = (1-\alpha)^{A_0} \quad , \quad A_0 = \frac{1-\alpha}{\eta+\alpha+(1-\alpha)\sigma} \quad , \quad A_{\zeta} = \frac{1+\eta}{\eta+\alpha+(1-\alpha)\sigma} \quad , \quad A_{\Psi} = A_0$$

$$(18b) \quad W_t = (1-\alpha)^{B_0} \zeta_t^{B_{\zeta}} \Psi_t^{B_{\Psi}} \quad , \quad \bar{W} = (1-\alpha)^{B_0} \quad , \quad B_0 = \frac{\eta+(1-\alpha)\sigma}{\eta+\alpha+(1-\alpha)\sigma} \quad , \quad B_{\zeta} = \frac{\eta+\sigma}{\eta+\alpha+(1-\alpha)\sigma} \quad , \quad B_{\Psi} = B_0 - 1.$$

By the Markov assumption (or log normality of (ζ_t, Ψ_t))

$$(18c) \quad (Y_t - \bar{Y}) = \bar{Y}A_\zeta(\zeta_t - 1) + \bar{Y}A_\psi(\psi_t - 1) \quad , \quad A_\zeta > 0, A_\psi > 0$$

$$(18d) \quad (W_t - \bar{W}) = \bar{W}B_\zeta(\zeta_t - 1) + \bar{W}B_\psi(\psi_t - 1) \quad , \quad B_\zeta > 0, B_\psi < 0.$$

These show the shock has the noted key property: *a positive ψ_t shock decreases the wage but increases output.*

Turning now to the profit function, note that due to the market structure of the purely competitive firm, a maximized real profit function is the same as (5b). This implies that although ψ_t impacts the dynamics of wages, it does not enter the profit function directly, the second order approximation is as in (9a)-(9b) and the derivatives of the maximized profit function are as in (11a)-(11b) with $\Pi_{W\psi} = 0$. The difference is only in the stochastic behavior of the competitive wage. I thus proceed to estimate the difference in expected profits:

$$\begin{aligned} \Delta_t(1) = & \frac{1}{2} \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1+\alpha}{\alpha}} E_t[(W_{t+1} - \bar{W})^2 - (W_t^e - \bar{W})^2] \\ & - \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1}{\alpha}} E_t[(W_{t+1} - \bar{W})(\zeta_{t+1} - 1) - (W_t^e - \bar{W})(\zeta_{t+1} - 1)] \end{aligned}$$

But now

$$E_t[(W_{t+1} - \bar{W})^2 - (W_t^e - \bar{W})^2] = \bar{W}^2 [E_t[B_\zeta(\zeta_{t+1} - 1) + B_\psi(\psi_{t+1} - 1)]^2 - [B_\zeta\lambda_\zeta(\zeta_t - 1) + B_\psi\lambda_\psi(\psi_t - 1)]^2] = \bar{W}^2 [B_\zeta^2\sigma_\zeta^2 + B_\psi^2\sigma_\psi^2]$$

$$E_t[(W_{t+1} - \bar{W})(\zeta_{t+1} - 1) - (W_t^e - \bar{W})(\zeta_{t+1} - 1)] = \bar{W}B_\zeta[\lambda_\zeta^2(\zeta_t - 1)^2 + \sigma_\zeta^2 - \lambda_\zeta^2(\zeta_t - 1)^2] = \bar{W}B_\zeta\sigma_\zeta^2.$$

Hence,

$$\Delta_t(1) = \frac{1}{\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\bar{W})^{-\frac{1-\alpha}{\alpha}} [B_\zeta\sigma_\zeta^2(\frac{1}{2}B_\zeta - 1) + B_\psi^2\sigma_\psi^2].$$

Now the conclusion is ambiguous: $B_\zeta\sigma_\zeta^2(\frac{1}{2}B_\zeta - 1) < 0$ but $B_\psi^2\sigma_\psi^2 > 0$! To overturn $\Delta_t(1) < 0$ and conclude that $\Delta_t(1) > 0$, parameters must satisfy

$$(19) \quad \frac{\sigma_\psi^2}{\sigma_\zeta^2} > \left(\frac{B_\zeta}{B_\psi^2}\right) \left(1 - \frac{1}{2}B_\zeta\right).$$

No real economy comes close to meeting condition (19)! Standard parameter values $\eta = 1, \sigma = 0.9, \alpha = \frac{1}{3}$ imply the expression on the right equals 16.82 which *requires σ_ψ^2 to be 17 times larger than the variance of the technology shocks.*

3.6 Monopolistic Competitive Market Structure

Does the argument extend to monopolistic competitive market structure used in the NK literature? To answer this question consider the simpler case of fully flexible prices where firms select optimal prices at all

dates. The more complex and lengthier case of sticky prices is avoided but I explain later why *adding sticky prices only strengthens the argument here*. In order to study the effect of a shock that does not provide a motive for an expected wage equilibrium I assume, as in Section 3.5, two shocks: one to productivity and one to the marginal utility of consumption. I start again with the fully flexible wage market.

The demand function for output of monopolistic competitive firm j is given by

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\theta} Y_t \text{ with implied required labor input of } N_{jt} = \left[\frac{1}{\zeta_t} \left(\frac{P_{jt}}{P_t}\right)^{-\theta} Y_t\right]^{\frac{1}{1-\alpha}}.$$

In equilibrium Y_t is total output. Every firm is also a household that carries out the optimization given its stochastic discount rate. Since $Y_{jt} = \zeta_t N_{jt}^{(1-\alpha)}$, j 's real marginal cost function is

$$\varphi_{jt} = W_t \frac{1}{(1-\alpha)\zeta_t} N_{jt}^\alpha.$$

For monopolistic competitors setting demand for labor is the same as maximizing over p_{jt} . The profit function is

$$\Pi_{jt} = \frac{1}{P_t} [p_{jt} Y_{jt} - W_t N_{jt}] = \left[\left(\frac{P_{jt}}{P_t}\right) Y_{jt} - W_t \left(\frac{Y_{jt}}{\zeta_t}\right)^{\frac{1}{1-\alpha}}\right] \text{ while } Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\theta} Y_t.$$

Profit maximization means

$$\frac{\partial \Pi_{jt}}{\partial p_{jt}} = (1-\theta) \left(\frac{P_{jt}}{P_t}\right)^{-\theta} \frac{Y_t}{P_t} - \varphi_{jt} \left[(-\theta) \left(\frac{P_{jt}}{P_t}\right)^{-\theta-1} \left(\frac{Y_t}{P_t}\right)\right] = 0 \Rightarrow \left(\frac{P_{jt}}{P_t}\right) = \frac{\theta}{(\theta-1)} \varphi_{jt}, \quad \theta > 1.$$

The optimal price function is

$$(20) \quad \left(\frac{P_{jt}}{P_t}\right) \equiv p^*(Y_t, W_t, \zeta_t) = \left\{ \frac{\theta}{(\theta-1)} W_t \frac{1}{(1-\alpha)\zeta_t} \left[\frac{1}{\zeta_t} Y_t\right]^{\frac{\alpha}{1-\alpha}} \right\}^{\frac{1}{1+\theta\frac{\alpha}{1-\alpha}}}.$$

In steady state $1 = \frac{\theta}{(\theta-1)} \bar{W} \frac{1}{(1-\alpha)} \bar{Y}^{\frac{\alpha}{1-\alpha}} = \left(\frac{\bar{P}_j}{\bar{P}}\right)$. I now ignore index j since by (20) all firms choose the same price.

I compare profits of a competitive firm in a flexible wage market with profits of a firm that deviates with expected wage offers. In order to enable a deviating firm to select an optimal price holding real wage fixed, the profit function is then written as a function of the optimal price

$$(21) \quad \Pi(Y_t, W_t, \zeta_t) = p^*(Y_t, W_t, \zeta_t)^{(1-\theta)} Y_t - W_t p^*(Y_t, W_t, \zeta_t)^{-\frac{\theta}{1-\alpha}} \left(\frac{Y_t}{\zeta_t}\right)^{\frac{1}{1-\alpha}}$$

with partial derivatives that show the profit function is convex with respect to the wage

$$(22) \quad \Pi_{p^*} = 0, \quad \Pi_W = -\left(p^*\right)^{-\frac{\theta}{1-\alpha}} \left(\frac{Y_t}{\zeta_t}\right)^{\frac{1}{1-\alpha}}, \quad \Pi_{WW} = \frac{\theta}{(1-\alpha)+\theta\alpha} \left(\frac{Y_t}{\zeta_t}\right)^{\frac{1}{1-\alpha}} \left(p^*\right)^{-\frac{\theta}{1-\alpha}} \frac{1}{W_t} > 0.$$

A firm controls neither productivity ζ_t nor demand Y_t , both of which are functionally related to the wage and to see this note that equilibrium wage is computed from the equilibrium condition $\left(\frac{P_{jt}}{P_t}\right) = 1$, consequently

$$W_t = (1-\alpha) \frac{(\theta-1)}{\theta} \zeta_t^{1-\alpha} Y_t^{-\frac{1}{1-\alpha}}.$$

Now recall optimal labor supply

$$(23) \quad W_t = (\psi_t)^{-1}(L_t)^\eta(C_t)^\sigma.$$

Equilibrium conditions $L = N$ and $C = Y$ imply

$$(24) \quad \left(\frac{Y_t}{\zeta_t}\right)^{\frac{\eta}{1-\alpha}}(Y_t)^\sigma = \psi_t(1-\alpha)\frac{(\theta-1)}{\theta}\zeta_t^{\frac{1}{1-\alpha}}Y_t^{-\frac{\alpha}{1-\alpha}}$$

hence in a flexible wage market

$$(25a) \quad Y_t = \left[(1-\alpha)\frac{(\theta-1)}{\theta}\right]^{\frac{1-\alpha}{\eta+\alpha+(1-\alpha)\sigma}}\zeta_t^{\frac{1+\eta}{\eta+\alpha+(1-\alpha)\sigma}}\psi_t^{\frac{1-\alpha}{\eta+\alpha+(1-\alpha)\sigma}}, \quad \bar{Y} = \left[(1-\alpha)\frac{(\theta-1)}{\theta}\right]^{\frac{1-\alpha}{\eta+\alpha+(1-\alpha)\sigma}}$$

$$(25b) \quad W_t = \left[(1-\alpha)\frac{(\theta-1)}{\theta}\right]^{\frac{\eta+(1-\alpha)\sigma}{\eta+\alpha+(1-\alpha)\sigma}}\zeta_t^{\frac{\eta+\sigma}{\eta+\alpha+(1-\alpha)\sigma}}\psi_t^{\frac{-\alpha}{\eta+\alpha+(1-\alpha)\sigma}}, \quad \bar{W} = \left[(1-\alpha)\frac{(\theta-1)}{\theta}\right]^{\frac{\eta+(1-\alpha)\sigma}{\eta+\alpha+(1-\alpha)\sigma}}.$$

The Markov distribution assumption or log normality of (ζ_t, ψ_t) imply

$$(25c) \quad (Y_t - \bar{Y}) = \bar{Y}A_\zeta(\zeta_t - 1) + \bar{Y}A_\psi(\psi_t - 1) \quad , \quad A_\zeta = \frac{1+\eta}{\eta+\alpha+(1-\alpha)\sigma} > 0, \quad A_\psi = \frac{1-\alpha}{\eta+\alpha+(1-\alpha)\sigma} > 0$$

$$(25d) \quad (W_t - \bar{W}) = \bar{W}B_\zeta(\zeta_t - 1) + \bar{W}B_\psi(\psi_t - 1) \quad , \quad B_\zeta = \frac{\eta+\sigma}{\eta+\alpha+(1-\alpha)\sigma} > 0, \quad B_\psi = \frac{-\alpha}{\eta+\alpha+(1-\alpha)\sigma} < 0.$$

Note that in (25d) shocks to marginal utility of consumption affect output and the wage *with opposite signs* and this condition strengthens firm incentive not to hedge against the volatility of such shocks.

In the flexible wage market a firm takes aggregate demand, the wage rate and exogenous shocks as given. Equilibrium demand is a function of the shocks hence it follows from (25a) that Y is as function of (ζ_t, ψ_t) . However, the deviating firm controls the wage which is kept as a separate variable. Again, I ignore the index j and use the notation $W_t^e \equiv E_t W_{t+1}$. Now expand the profit function up to second order around steady state values $(\bar{\Pi}, \bar{W}, \bar{Y}, \bar{\zeta}, \bar{\psi})$ and take the difference between the flexible wage firm and the expected wage firm to deduce

$$\Delta_t(1) = \frac{1}{2}\Pi_{ww}E_t[(W_{t+1} - \bar{W})^2] + \Pi_{w\zeta}E_t[(W_{t+1} - \bar{W})(\zeta_{t+1} - 1)] + \Pi_{w\psi}E_t[(W_{t+1} - \bar{W})(\psi_{t+1} - 1)] \\ - \left[\frac{1}{2}\Pi_{ww}[(W_t^e - \bar{W})^2] + \Pi_{w\zeta}E_t[(W_t^e - \bar{W})(\zeta_{t+1} - 1)] + \Pi_{w\psi}E_t[(W_t^e - \bar{W})(\psi_{t+1} - 1)]\right].$$

Steady state values of the partial derivatives of (21) given the optimal price function (20) and keeping in mind that Y is taken here as known function of the shocks as in (25a)

$$\Pi_{ww} = \frac{\bar{Y}^{\frac{1}{1-\alpha}}}{(1-\alpha+\alpha\theta)}\frac{\theta}{\bar{W}}, \quad \Pi_{w\zeta} = -\frac{\bar{Y}^{\frac{1}{1-\alpha}}}{(1-\alpha+\alpha\theta)}\left[(\theta-1) + \frac{(1+\eta)}{(\eta+\alpha+(1-\alpha)\sigma)}\right], \quad \Pi_{w\psi} = -\frac{\bar{Y}^{\frac{1}{1-\alpha}}}{(1-\alpha+\alpha\theta)}\left[\frac{1-\alpha}{(\eta+\alpha+(1-\alpha)\sigma)}\right]$$

and

$$(26) \quad \Delta_t(1) = \frac{\bar{Y}^{\frac{1}{1-\alpha}}}{(1-\alpha+\alpha\theta)}\bar{W}\left[\sigma_\zeta^2 B_\zeta\left[\frac{\theta}{2}B_\zeta - \left(\theta-1 + \frac{1+\eta}{(\eta+\alpha+(1-\alpha)\sigma)}\right)\right] + \sigma_\psi^2 B_\psi\left[\frac{\theta}{2}B_\psi - \frac{1-\alpha}{(\eta+\alpha+(1-\alpha)\sigma)}\right]\right].$$

The first term in (26) (due to ζ) is negative and the second (due to ψ) is positive. For realistic economies with parameter values $\alpha = \frac{1}{3}$, $\eta = 1$, $\sigma = 0.9$, $\theta = 6$ the expression in (1.26) satisfies

$$(27) \quad \left[\sigma_{\zeta}^2 B_{\zeta} \left[\frac{\theta}{2} B_{\zeta} - \left(\theta - 1 + \frac{1 + \eta}{(\eta + \alpha + (1 - \alpha)\sigma)} \right) \right] + \sigma_{\psi}^2 B_{\psi} \left[\frac{\theta}{2} B_{\zeta} - \frac{1 - \alpha}{(\eta + \alpha + (1 - \alpha)\sigma)} \right] \right] = -6.8796 \sigma_{\zeta}^2 + 0.1486 \sigma_{\psi}^2.$$

Again, (27) explains why for all real economies $\Delta_t(\mathbf{1}) < 0$ and the firm has an incentive to hedge labor input. For (27) to be positive we must have $\sigma_{\psi}^2 > 47 \sigma_{\zeta}^2$. Finally, the argument for longer horizon is similar to the one presented earlier. I then conclude

Proposition 3: Under a monopolistic competitive market structure and for any horizon, if $\sigma_{\psi} = 0$ it is optimal for the firm to offer and for the workers to accept an expected wage which is averaged as in (15). If $\sigma_{\psi} > 0$ the proposition remains true for all empirically relevant parameter values of the economy.

The conclusion is then the same as in the case of a competitive market.

3.7 The Contributing Effect of Sticky Prices

Sticky prices weaken the ability of a monopolistically competitive firm to gain from fluctuating input prices. If such a firm cannot change its price, the profit function becomes

$$\Pi_{jt} = \frac{1}{P_t} [p_{j,t-1} Y_{jt} - W_t N_{jt}] = \left[\left(\frac{p_{j,t-1}}{P_t} \right) Y_{jt} - \frac{W_t}{P_t} \left(\frac{Y_{jt}}{\zeta_t} \right)^{\frac{1}{1-\alpha}} \right] \text{ while } Y_{jt} = \left(\frac{p_{j,t-1}}{P_t} \right)^{-\theta} Y_t.$$

which is a linear function in the wage, not convex. Such a firm has an increased benefit from an expected wage offer since it may not be able to match wage variability with changes in its price. This explanation views sticky prices as a further *cause for sticky wages*, in conflict with the literature cited earlier that explains sticky prices by non-competitive sticky wages. No consensus seem to have emerged, as yet, on the cause and degree of inflexible prices. The original explanation (see Mankiw (1985), Akerlof and Yellen (1985)) was that sticky prices result from “menu cost” which are real cost of posting price changes. Subsequent empirical work (e.g. Bils and Klenow (2004), Hosken and Reiffen (2004), Nakamura and Steinsson (2008), Kehoe and Midrigan (2010), Eichenbaum, Jaimovich, and Rebelo (2011)) questioned the assumed price inflexibility and revealed that price changes exhibit complex patterns. For example, sales express price flexibility but *short term* price changes followed by reversion to the pre-sale prices is a form of some price inflexibility.

In sum, although wage stickiness contributes to price stickiness the ideas developed here imply wage stickiness and price stickiness are very different phenomena. Accordingly, Taylor’s (1979), (1980), (1999) use

of union contracting to explain wage stickiness is not compatible with the empirical reality of relatively weak labor unions and is actually contradicted by the evidence in Bewley (2009) which suggests unions support price flexibility. However, the model of *staggered wages* is entirely compatible with the empirical evidence and with an expected wage equilibrium explored here.

4. Expected Wage Equilibrium: Emergence of Competition and Involuntary Unemployment

Up to now the analysis involved a game of only one firm and its workers, given a functioning flexible wage equilibrium labeled as “imaginary” since this market ultimately closes when all firms end following the same optimal strategy. It is instructive to consider the question of equilibrium perfection first with only one firm. This is substantially changed when all firms follow the same strategy and the flexible wage market closes.

4.1 Perfection of a one firm equilibrium

Faced with a lower flexible wage at some future date, would the firm abandon the expected wage equilibrium and lower its wage and hire workers to work for a lower wage? Such violation of the firm’s wage offer will be taken as a permanent abandonment of the equilibrium, entailing significant direct cost. Some are central to the search and matching literature. I list them briefly as follows:

- (i) Loss of investment cost of hiring the old employees and direct investment in hiring new employees;
- (ii) Loss of past investments in developing all senior workers;
- (iii) Abrupt change of the labor force cause delays and expensive temporary inefficiencies in production;
- (iv) Some (e.g. Solow (1979), Akerloff (1982), Akerloff and Yellen (1988),(1990) and Bewley (1999)) propose the new employees would be less efficient since lower wages reduce labor efficiency.

Although these are important factors, the theory developed here suggests that far more important are the lost present value of the long term gains to the firm from the institutional arrangement of the expected wage equilibrium. To see this note that faced with a lower wage in the flexible wage market the firm can maintain its credibility by keeping its offered wage *for the duration of the offer* and revising it at the end of that period when new offers are made. At that time the wage scale will align the expected wage better to the flexible wage. I have already pointed out that an equilibrium with infinite horizon offers cannot be perfect hence a constant wage is not a perfect equilibrium (see Hall (2005a), (2005b)). Restricted to a duration of one period expected wage offers can sustain a perfect equilibrium since at the end of each period a new offer is made for the next period and the gain from a one period deviation from the expected wage cannot compensate the firm for loss of an infinite

sequence of future gains from the expected wage equilibrium. For any duration, the strategy of keeping its promise cost the firm the forgone capitalized profits that it could have earned in the flexible wage market during the period covered by an offer. Alternatively, it can abandon the equilibrium and gain the extra capitalized profits in the flexible wage market, taking into account the persistence of lower wages that may last longer than the offer's duration. However, abandoning the equilibrium entails accepting the much larger losses consisting of the present value of all future higher expected profits and higher utility of risk averse firm's owners at the infinite number of future dates when expected wage equilibrium would offer the firm a superior alternative. Due to the shocks' Markov structure the flexible wage reverts to steady state geometrically hence the value of a short duration gain from a lower wage in the flexible wage market is much smaller than the value of all future gains from the expected wage equilibrium. An optimal duration weights cost against gains and selects a length of time that maximizes expected net firm's utility gains and which can be sustained as a perfect equilibrium. I do not develop in this paper exact conditions for such optimal duration and perfection since my approach is empirically oriented and it is a fact that the established convention is of an equilibrium with offer duration of four quarters.

The above is an argument to explain why firms do not lower regular wages in recessions when irregular wages do adjust. The gain from lowering wages for a recession's short duration are smaller than the value to the firm of the long term gains from a very desirable equilibrium structure. This is also the answer to Barro's (1977) criticism that is based on a purely atomistic view of competition under which each actor is on his own. This view is correct if complete insurance markets are available. However, it fails to recognize the fact that when markets are incomplete competition create new institutions that compensate for markets that cannot exist due to incentives or externalities. In the broad sense of dynamic adaptation, an expected wage equilibrium with inflexible wage is as "competitive" as any competitive economy but with different adjustment mechanism. Such mechanism incorporate more quantity adjustments that are as vital as price adjustment is to the traditional view of equilibrium but with different implications to efficiency and public policy.

4.2 *The limit equilibrium*

As the number of firms that adopt the expected wage strategy increases, the environment changes so that some might suggest the imaginary economy does not matter. After all without a flexible wage workers have no alternative to expected wage offers and job arbitrage between such offers and the flexible wage market is not available. Also, deviation of one firm from the flexible wage market creates economic rents with capital values to the firm and workers and such rents render valuable the priority in future job offers given existing employees.

As the number of deviating firms increases, competition emerges among firms who seek workers but do not compete with flexible wages. The instrument of competition are inflexible wage scales which exhibit little response to current economic conditions giving rise to a new equilibrium with inflexible wage offers. It results in the emergence of involuntary unemployment as a real phenomenon of inefficiency. With new competition among firms and involuntary unemployment, what of the imaginary economy is then relevant?

The Emergence of Competition and Involuntary Unemployment. Why “Expected” wage and not other constant wages? In a limit economy firms compete with wage scales and although conditions of perfection are altered, the equilibrium is not that different. Multiple equilibria may exist, but *expected wage offers* are compatible with the long term flexible wage equilibrium. To see why, allow an out of equilibrium, no perfect memory adjustment stage that lets firms experiment with wage scales and let workers shop for jobs. Expected wage offers trace closely the flexible wage but with lower volatility. For 1 quarter duration in the economy with a single technological shock the offer at t is

$$(W_t - \bar{W}) = \frac{1}{\lambda_\zeta} (W_t^e(1) - \bar{W}).$$

With $\lambda_\zeta < 1$ the flexible wage fluctuates around its mean which is the wage scale. Since $(W_t - \bar{W})$ clears labor market *at each t*, $(W_t^e(1) - \bar{W})$ clears it *on average over time*. The flexible wage is the anchor that changes with labor productivity and so does the wage scale but with lower volatility. In the long run, firms with higher wage scales go bankrupt and those with lower scales cannot find workers. As memory and option values are introduced, duration rises and 4 quarter duration offers are very similar since they take the form

$$(28) \quad W_t^e(4) - \bar{W} = \frac{1}{4} (\lambda_\zeta + \lambda_\zeta^2 + \lambda_\zeta^3 + \lambda_\zeta^4) [W_t - \bar{W}]$$

but with lower volatility since each offered wage at t *remains constant at t+1, t+2, t+3 and t+4*. In short, the long term behavior of a wage scale is correlated with that of the flexible wage but with the same mean. Hence, it replicates the long term market clearing properties of the flexible wage but the longer are wage offers’ duration, the less responsive it is to current state which is exactly the inflexibility observed in the data.

The resulting competitive equilibrium gives rise to a market institution of mutual insurance against wage volatility but inflexible wages result in the emergence of socially inefficient involuntary unemployment. Failure of competition to attain Pareto Optimality in incomplete markets is not surprising and this paper is motivated by the fact that Democracy may fail to find a correcting mechanism. The conclusion that inflexible wages benefit a firm and its employed workers while the unemployed *pay the implied cost* is compatible with Bewley’s (1999) finding that management and employed workers oppose lower wages in recessions while unions support it. It also explains why in highly unionized societies such as Germany, unemployment is often shared by a collective

reduction in hours instead of leaving some members without any work.

Inflexible wage offers are entirely legal and do not violate antitrust laws. Purists who believe an atomistic economy with flexible prices is a *required condition* for individual liberty may object to inflexible wages on ideological ground but will find no fault in the spontaneous emergence of this voluntary institution created by inventive market competition. I now translate the abstract theory developed here to an actual wage setting which can be incorporated explicitly into a model economy.

5. The Theory in Practice: the Wage Adjustment Process

How does the expected wage theory work in practice? I start with a word of caution. Actual labor markets are far more complex than the simple set up in the models of the previous chapter, rendering a wage setting mechanism that does not result from actions of an impersonal auctioneer or a traditional invisible hand only a broad approximation of reality. That is, it is important to keep in mind that wage setting is an outcome of complex human interactions in a world where labor is not a homogenous input, human talents and abilities are different, information about ability is asymmetric, workers with different skills and different experience make different contribution and humans hold diverse beliefs about future events and have different aversion to risk. Moreover, a firm's employment of a worker with large human capital entails a working collaboration of real and human capital and as the capital components of both sides grow, it becomes more important to coordinate the activities of the two inputs, both of which seek high returns but desire them to be as riskless as possible. Both seek to avoid fluctuating returns and the destructive depreciation that results from unemployment.

The implication is that a useful theory of wages can only explain the principles underlying the process of wage setting and how this process interacts with the rest of the economy, helping to explain the behavior of economic aggregates. This objective cannot benefit too much from examining in detail the mechanism of job search or the implementation of the wage schedule by each firm. Indeed, there is limited value in any detailed microeconomic theory of wage setting since only general principles would ultimately explain the behavior of aggregates. The theory developed in the previous sections was confined to real wage offers made to long term regular employees because this relation and its wage setting is *the essential factor that drives the behavior of all market wages*. Such a formal treatment clearly abstracts from several important problems that exist in practice and need to be addressed in order to make it useful in applications. The wage adjustment mechanism developed here translates the previous conclusions into a wage setting model that can then be used in a formal analysis. To accomplish this I take the results of earlier sections as a starting point which is modified in response to

qualifications to be discussed. In carrying out these modifications my approach is empirically based and practical, avoiding abstract deductive procedures. Before discussing the qualifications I further clarify the concepts of a *wage scale*. I will continue to use the notation of earlier sections where all wages are real wages.

5.1 The Wage Scale and the Reference Wage of Regular Employees

The *wage scale* for offer duration of four quarters, which is my canonical case, defined earlier by $\hat{w}_t^e = (1/4)E_t[\hat{w}_{t+1}^F + \hat{w}_{t+2}^F + \hat{w}_{t+3}^F + \hat{w}_{t+4}^F]$ where for any X , $\hat{x}_t = (X_t - \bar{X}) / \bar{X}$. This time I denote by \hat{w}_t^F the flexible wage and for the rest of the paper W_t is equilibrium *inflexible* real wage. In the flexible wage economy \hat{w}_t^F is a linear function of the state but evidence suggests labor productivity is the primary factor. Figure 2 shows mean wage traces productivity per man hour very closely *when the real wage is computed with the PPI, not the CPI*.

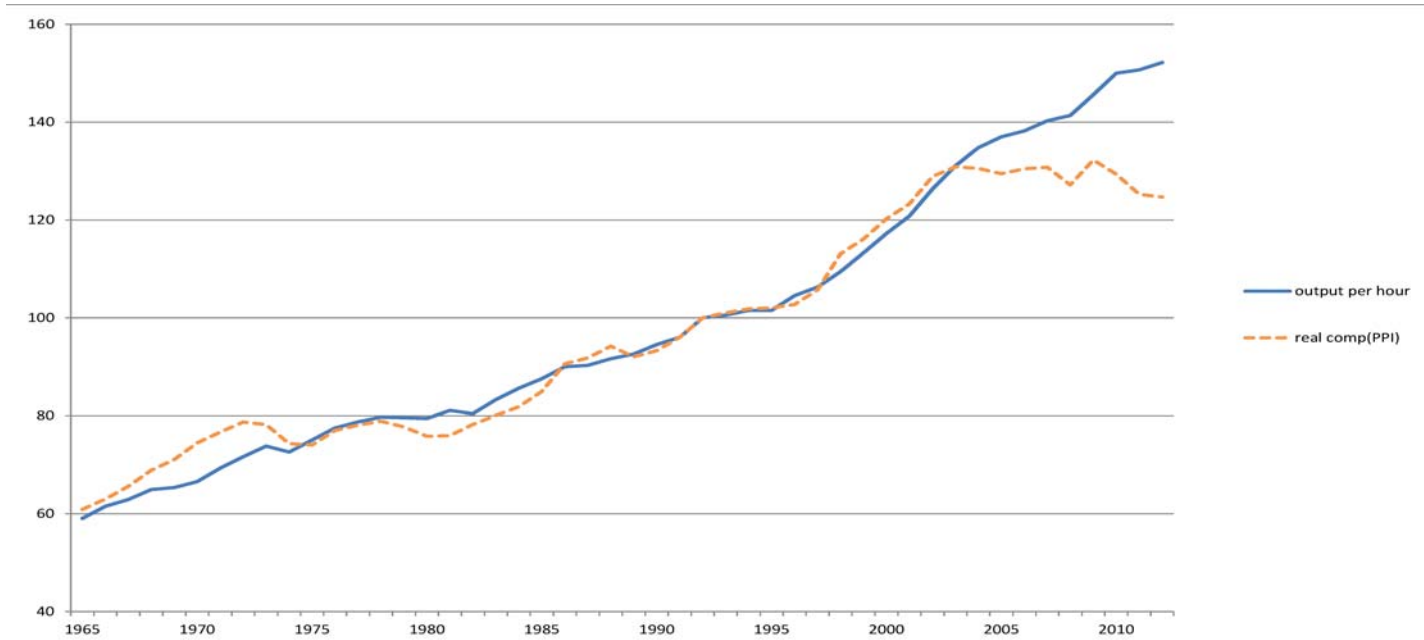


Figure 2: Output per man hour is for the US business sector from Table B-49 of the Economic Report of the President, 2014. Real Compensation (PPI) is total compensation per hour in the business sector using the PPI. Table B-49 of the same report where the nominal total compensation is divided by the producer price index for all finished good, Table B-65 of the same report. All indices are scaled at 1992 =100

It is well known CPI based real wages have been stagnant. Further deterioration in the labor market was caused by internal polarization (see, Autor and Dorn (2013), Autor (2014)). However, the issue here is the relation between productivity and real wage offers based on *optimal inputs by firms*. A rational firm considers relative cost of labor to other inputs hence the PPI is the appropriate index and Figure 2 shows deviations in the

tight relation of mean wage with productivity occurred at the height of the Viet Nam war and during the elimination of corporate pensions and other benefits that started around 2004. Such factors can be accounted for later, but apart from them the only fluctuating factor is the expected real contribution of a regular employees to the firm's output, approximated by

$$(29) \quad \hat{w}_t^e = \left(\frac{1}{4}\right) A_{wF}^\zeta E_t[\hat{\zeta}_{t+1} + \hat{\zeta}_{t+2} + \hat{\zeta}_{t+3} + \hat{\zeta}_{t+4}] = \left(\frac{1}{4}\right)(\lambda_\zeta + \lambda_\zeta^2 + \lambda_\zeta^3 + \lambda_\zeta^4) A_{wF}^\zeta \hat{\zeta}_t$$

where A_{wF}^ζ is the elasticity of the competitive real flexible wage with respect to productivity and $\hat{\zeta}_t = \zeta_t - 1$.

The offered wage (29) becomes a constant wage for four *quarters* hence, at any time there are four regular wages in the market: $(W_{t-1}^e, W_{t-2}^e, W_{t-3}^e, W_{t-4}^e)$. Since I study aggregate behavior with log-linear approximation, wage cost are additive hence it is the mean wage with an effect on aggregate variables. I then define the *Reference Wage* to be the mean market wage of regular employees

$$(30a) \quad W_t^* = [W_{t-1}^e W_{t-2}^e W_{t-3}^e W_{t-4}^e]^{\frac{1}{4}}.$$

Taking percentage deviations from steady state

$$(30b) \quad \hat{w}_t^* = \frac{1}{4}[\hat{w}_{t-1}^e + \hat{w}_{t-2}^e + \hat{w}_{t-3}^e + \hat{w}_{t-4}^e] = B_w^\zeta (\hat{\zeta}_{t-1} + \hat{\zeta}_{t-2} + \hat{\zeta}_{t-3} + \hat{\zeta}_{t-4}) \quad , \quad B_w^\zeta = \left(\frac{1}{16}\right)(\lambda_\zeta + \lambda_\zeta^2 + \lambda_\zeta^3 + \lambda_\zeta^4) A_{wF}^\zeta.$$

Figure 2 shows persistent random deviations from wage scales may occur, implying the reference wage may contain a random term ρ_t^{Rw} and hence be written as

$$\hat{w}_t^* = B_w^\zeta (\hat{\zeta}_{t-1} + \hat{\zeta}_{t-2} + \hat{\zeta}_{t-3} + \hat{\zeta}_{t-4}) + \rho_t^{Rw}$$

but since this represent special transitory forces, I generally ignore it here.

With an explicit scale (29) as the basis of wage setting, I turn to the practical problems noted earlier. If one applies the same approach as (29) to all jobs, the wage scale of a firm with J jobs is a J dimensional vector of functions $(\hat{w}_t^{e1}, \hat{w}_t^{e2}, \dots, \hat{w}_t^{eJ})$ which are rules of making offers at date t for regular date t+1 employment in the J jobs. Time variability of a rule reflects changes in productivity trends over time or change in the contribution of a job to the firm. The rules may also be changed due to changed market conditions. A multi-job wage scale is central to an employee's long term relation with the firm since it is the internal wage escalator that offers future wage increases as the employee gains experience and status within the firm and such prospects, in addition to an offer of known wage, are the desirable properties of regular employment. Although not explicit in (29), it is in the background as an essential component of the wage offer to regular employees. In contrast, the scale in (29) specifies a rule of job offers made only to a single type of labor. This is so because there is little qualitative difference in the process of regular wage offers between two different jobs and our search for a characterization of wage inflexibility is not advanced by studying this heterogeneity.

The distinction between “regular” and “irregular” employment is the first qualification to (29). I do not

focus on the difference in wage setting between a secretary and a librarian but instead, on the difference between an electrical engineer holding a regular job with expectation of long term employment vs. an electrical engineer hired for temporary work, or for only one project without any relation to the firm. These categories are based on employment relation to the firm and entail different job descriptions, discussed further in assessing wage setting of irregular employees. It results in different wage settings that lead to *different wage responses to unemployment* and this is the distinction sought here. In short, different jobs surely pay different wages but this is a study of wage inflexibility hence *I focus on differences among jobs in terms of their degree of wage inflexibility to current economic conditions*. The finding in earlier sections is that wages of regular employees exhibit small response to current conditions and this is the main feature of wages based on (29). By (30a) this slow changing property is inherited by the reference wage.

The second key qualification to (29) is the complex problem presented by inflation. The development so far specifies wages in real terms and (29) is such a wage scale. Hence, up to now I assumed a full indexation of wages and this is clearly not in accord with the empirical evidence. Wages are specified in nominal terms and inflation has a complex impact on the wage setting process. This is a well known difficult and controversial problem which is related to the basic properties of the Phillips Curve and discussed later.

Before modifying (29) I note that the term *reference wage* conveys the idea of a rule used in all wage settings. It actually averages over 8 quarters: a scale is an average over 4 *future* quarters and a reference wage averages over four *past* quarters. Models of staggered wages imply aggregate variables such as inflation depend on a distributed lag of past wages (e.g. Fischer (1977), Taylor's (1999), Gertler and Trigari (2009)). My approach is compatible with theirs but I focus instead, on the *lag structure of the shocks*. To see the empirical implication of inflexibility note that in a typical flexible wage model with $\lambda_{\zeta}=0.90, \eta=1, \sigma=0.9, \alpha=(1/3)$, the equilibrium elasticity is $A_{wF}^{\zeta}=0.98$ hence $(\lambda_{\zeta}+\lambda_{\zeta}^2+\lambda_{\zeta}^3+\lambda_{\zeta}^4)=3.0952$ and $B_w^{\zeta}=(1/16)(0.98)(3.0952)=0.19$. It means a 1% change in $\hat{\zeta}_t$ changes the flexible wage by 0.98% but the t+1 reference wage changes only by 0.19% but continues to change slowly as future scales adjust. Finally, other factors will be shown to impact mean wage, making it endogenous variable, different from the reference wage. In practice I deduce A_{wF}^{ζ} from the equilibrium map of *the same model with a flexible wage*. In an environment with growth, the reference wage and wage scales measure expected trend productivity. For economies without growth it can be computed analytically but in an economy with growth the problem is somewhat more involved hence one first computes the competitive wage of the economy with flexible wages and then take average expected wage deviations from trend due only to output shocks *ignoring the effect of other state variables*. Hence, approximating wage scales by estimating trend

productivity is the norm, although differences across firms, jobs and occupations naturally exist in real markets.

If all employees were regular long term employees the mean market wage would have been the reference wage. I now address the questions of irregular work and inflation.

5.2 *Setting Irregular Wages*

Empirical evidence deduced from data on the same individual over time shows wages of newly hired workers in entry level status are more responsive to unemployment than wages of regular employees. (e.g. Bils (1985), Keane, Moffitt, and Runkle (1988), Solon, Barsky, and Parker (1994), Pissarides (2009) and references there). A weaker response is reported in recent work on downward nominal wage rigidity (e.g. Benigno and Ricci (2008), (2011), Daly and Hobijn (2013)). It is also well established that wages paid for irregular job categories are more responsive to current economic conditions than wages paid for regular jobs. This includes wages for personal services, hospitality jobs, part time jobs and generally, short duration jobs. These cover categories such as temporary jobs, seasonal agricultural jobs, jobs on fixed duration projects like oil drilling or construction and investment projects. Irregular jobs are also filled by highly paid consultants or professionals hired for specific projects who often work under the legal status of self employment. It is well known that irregular jobs tend to be lower paying and lower skilled jobs while jobs that require high skill and heavy human capital investments are typically regular jobs. This is not accidental and is implied by my theoretical perspective that stresses *the joint incentives of the firm and its regular employees to reduce the risk of the return on their respective real and human capital*. Such incentives are stronger as the level of real and human capital invested rises. However, no occupational heterogeneity is postulated as I assume all workers do the same work.

How would the market set the wage of irregular jobs relative to their regular counterparts when a regular “counterpart” is understood to do the same physical work but with different relation to the firm? The answer is given by the nature of “irregular” employment. A regular bus driver drives a bus in the same way a temporary driver does, but there is a difference. Regular employees are loyal to the firm, are known and trusted by the firm to perform to the best of their ability and serve the firm’s interests. Irregular employees have no such incentives and are thus more risky to the firm. Mean productivity of all workers is assumed the same but it is recognized the productivity of irregular workers is more risky and they are given less responsibility. Some insist a characteristic of irregular workers is a high level of asymmetric information about their ability, but information asymmetry is just a component of the firm’s risk. Information is central in the case of entry level jobs, classified as irregular jobs, since irregular employees can become regular by improved information. Apart from entry level jobs all

irregular jobs are high turnover jobs by their own nature and short duration employment makes it impossible for such workers to have a long term relation with the firm and benefit from the firm's job and wage escalator. This has deep impact on their *work motives, incentives and dedication to the job and firm*. This motivation problem is the main source of the firm's risk. In short, I view irregular employees as doing the same work as regular employees but at a higher level of risk to the firm.

The above argument explains why the wage of regular employees is the cornerstone for setting the wage of irregular employees. Indeed, the two wage rates are assumed to be the same with two differences that respond to current conditions. First, it is responsive to the involuntary rate of unemployment which reflects current economic conditions. Second, this wage is assumed to be fully indexed and hence as a nominal wage it fluctuates with current inflation. The manner in which the wage responds to these two factors is shortly explained.

To introduce unemployment I need to anticipate, without detailed comments, the way unemployment is formulated in models of later chapters. The point which is relevant to the discussion here is the assumption that search unemployment results in a constant natural rate ρ of unemployment anticipated by job offers. Hence suppose at date t employment desired by the firms is N_t but, expecting that a net fraction ρ of jobs accepted will end due to quits and bad matches, firms offer $(1+\rho)N_t$. Total work supplied by labor is L_t hence the involuntary unemployment rate is

$$(38) \quad u_t = 1 - (1+\rho)\frac{N_t}{L_t}.$$

It is technically simpler to make the unrealistic but inconsequential assumption that search continues to take place in steady state so that in steady state $\bar{u} = 0$ and $(1+\rho)\bar{N} = \bar{L}$. With this clarified I now return to discuss wage setting of irregular workers.

To implement the conclusion in the discussion above let (W_t^R, W_t^{IR}) be the mean wage of regular and irregular employees, respectively, then their response to involuntary unemployment is expressed by

$$(39a) \quad W_t^R = W_t^*$$

$$(39b) \quad \frac{W_t^{IR}}{\bar{W}} = \left[\left((1+\rho)\frac{N_t}{L_t} \right)^\mu \frac{W_t^*}{\bar{W}} \right] e^{\rho_t^w}.$$

Since $(1+\rho)\bar{N} = \bar{L}$ and $\bar{W}^* = \bar{W}$ it follows that in steady state all wages are the same. The mean wage of irregular workers responds to the rate of unemployment with an elasticity μ discussed in the excellent surveys of Abraham and Haltiwanger (1995) and Pissarides (2009). These exhibit estimates of μ that vary widely with data used, estimation method and hence their implied limitations. Focusing on entry level wages which Pissarides (2009) defines as the wages of new matches, he selects a value of $\mu = 3.0$ which implies that an unemployment

rate of 9% would lower entry level wages by 27%! The 2008-2014 data do not exhibit such a large fall in wages of irregular workers. Indeed, since this recent data are compatible with the more conservative among the empirical estimates cited, I set this parameter at the modest value of $\mu = 1$ and since I take as a reference a natural unemployment rate of 5% due to search, this parameter value implies that a 9% unemployment rate entails an *involuntary* unemployment rate of 4% and that would drive the wage of irregular workers down by 4%. It will be seen in a moment that such a decline in wage of irregular workers entails a much lower reduction of mean wage in the economy. The added random component $e^{\rho_t^w}$ indicates the wage paid for irregular employment may be subject to random shocks that change such factors as the weather, immigration rules, minimum wage policy, etc.

If the proportion of regular employees in the labor force is denoted by a_w (estimated to be about 0.85) and the involuntary unemployment rate by u_t then the mean wage is

$$(40a) \quad W_t = (W_t^*)^{a_w} \left[\left((1 + \rho) \frac{N_t}{L_t} \right)^\mu W_t^* e^{\rho_t^w} \right]^{(1-a_w)}$$

hence

$$(40b) \quad \hat{w}_t = \hat{w}_t^* - \lambda_u u_t + (1 - a_w) \rho_t^w \quad u_t = \hat{\ell}_t - \hat{n}_t, \quad \lambda_u = \mu(1 - a_w).$$

Flexibility of irregular wages means the mean wage also responds to unemployment. Given the estimated values of $(a_w = 0.85, \mu = 1)$ discussed above, one has $\lambda_u = \mu(1 - a_w) = 1.0 \times 0.15 = 0.15$ but the evidence suggests that μ hence λ_u may change due to factors such as the degree of unionization, market structure, minimum wage or other political conditions. Consequently, it has been difficult to establish a stable empirical relation between time series of wages and unemployment. These parameter estimates imply that if unemployment rises to the very high rate of 9% (i.e. $u_t = 4\%$), the mean wage in the economy will decline by only 0.60% which is consistent with the evidence. All the decline will occur in wages of irregular jobs.

5.3 The Problem of Inflation

Adjustment of wages to inflation is a perplexing and controversial problem. The controversial part, debated extensively during the 1970's, is about the short and long run Phillips curves where the difference between these two curves is defined by the speed at which the real wage adjusts to inflation. In the long run the real wage fully adjusts to inflation, implying a “vertical” Phillips Curve. The question studied here is the short term response of the real wage to inflation fluctuations.

An assumption of short term full indexation is in conflict with the perspective of earlier sections as it entails *asymmetry in risk bearing* between workers and the firm. In an economy with sticky prices full

indexation means workers face no inflation risk while the firm faces the risk of being unable to adjust its own price in response to unexpected changes in the general price level while being obligated to bear the cost of adjusting the nominal wage of workers. Firms differ in willingness to index wages not only due to differences in stickiness of their own prices but also because of differences in correlation between their prices and the price level. Indexation is also affected by unionization and cohesion among workers since inflation is a common problem, not like a compensation problem of any one worker. High degree of cohesion is more likely to draw a response from a firm concerned with workers' moral. Such considerations explain why offers of fully indexed real wages are rare.

Offers are then made by firms in nominal terms and accepted in such terms by workers but both sides recognize the real value of an offer will be changed by future inflation. Since with incomplete markets there is no market for insurance against inflation risk, the two sides cannot carry out the same mutual wage insurance as they did in Earlier sections and must bear the risk of inflation. The theory in Earlier sections does not offer a solution to the problem of allocating the cost of inflation risk. Since such cost are related to risks a firm faces in not being able to adjust its price, such cost are likely to be idiosyncratic to each firm and its industry. Also, echoing the debate about long term vs. short term Phillips Curve, high inflation rates increase frequency of wage adjustments with complicating implications. For these reasons there is little to be gained from a formal modeling of the problem of sharing inflation cost. Instead I turn to the simple practical solution of using the empirically observed normal response of the parties to the presence of inflation. Before reviewing this evidence, it is useful to gain some insight from exploring the theoretical implications of no indexation at all.

5.4.1 *Mean Real Wage Under No Indexation*

Suppose a firm computes the expected real wage as before, but offers it in nominal form at date t without any advanced allowance for inflation thus adopting the policy of making all inflation adjustments at the date when the next wage offer is made. If t is the offer date for employment at $t+1$, $t+2$, $t+3$ and $t+4$ then the actual real wage $W_{t+j}^{(t)}$ paid at $t+j$, $j = 1,2,3,4$ is defined by

$$P_{t+j} W_{t+j}^{(t)} = P_t W_t^e \quad \text{for } j = 1,2,3,4.$$

The notation identifies date t when the wage offer was made. Real wages can then be expressed in the form

$$W_{t+j}^{(t)} = \left[\left(\frac{P_t}{P_{t+1}} \right) \dots \left(\frac{P_{t+j-1}}{P_{t+j}} \right) \right] W_t^e \quad \text{for } j = 1,2,3,4.$$

A fractional indexation at each date can also be expressed by

$$W_{t+j}^{(j)} = \left[\left(\frac{P_t}{P_{t+1}} \right) \dots \left(\frac{P_{t+j-1}}{P_{t+j}} \right) \right]^v W_t^e \quad \text{for } j = 1, 2, 3, 4, \quad v < 1.$$

Hence, apart from dates when new offers are made, the real wage paid has a recursive form

$$(41) \quad W_{t+j}^{(j)} = \left(\frac{P_{t+j-1}}{P_{t+j}} \right) W_{t+j-1}^{(j)} \quad \text{for } j = 1, 2, 3, 4.$$

Now suppose all firms follow the same policy, then there are four groups of firms paying different real wages, depending upon the date of their offer. At date $t+1$ actual real wages paid are then

$$(42a) \quad \text{Group 1: real wage 1 date after offer} \quad W_{t+1}^{(1)} = \left(\frac{P_t}{P_{t+1}} \right) W_t^e$$

$$(42b) \quad \text{Group 2: real wage 2 dates after offer} \quad W_{t+1}^{(2)} = \left(\frac{P_{t-1}}{P_t} \right) \left(\frac{P_t}{P_{t+1}} \right) W_{t-1}^e$$

$$(42c) \quad \text{Group 3: real wage 3 dates after offer} \quad W_{t+1}^{(3)} = \left(\frac{P_{t-2}}{P_{t-1}} \right) \left(\frac{P_{t-1}}{P_t} \right) \left(\frac{P_t}{P_{t+1}} \right) W_{t-2}^e$$

$$(42d) \quad \text{Group 4: real wage 4 dates after offer} \quad W_{t+1}^{(4)} = \left(\frac{P_{t-3}}{P_{t-2}} \right) \left(\frac{P_{t-2}}{P_{t-1}} \right) \left(\frac{P_{t-1}}{P_t} \right) \left(\frac{P_t}{P_{t+1}} \right) W_{t-3}^e$$

Now define the mean real wage in the economy by

$$W_{t+1} = [W_{t+1}^{(1)} W_{t+1}^{(2)} W_{t+1}^{(3)} W_{t+1}^{(4)}]^{\frac{1}{4}}.$$

Use (42a)-(42d) to deduce

$$W_{t+1} = [W_t^e W_{t-1}^e W_{t-2}^e W_{t-3}^e]^{\frac{1}{4}} \left[\left(\frac{P_t}{P_{t+1}} \right) \left(\frac{P_{t-1}}{P_t} \right)^{\frac{3}{4}} \left(\frac{P_{t-2}}{P_{t-1}} \right)^{\frac{2}{4}} \left(\frac{P_{t-3}}{P_{t-2}} \right)^{\frac{1}{4}} \right].$$

Now use (30a) to have

$$(43) \quad \hat{w}_{t+1} = \hat{w}_{t+1}^* - \left[\hat{\pi}_{t+1} + \frac{3}{4} \hat{\pi}_t + \frac{2}{4} \hat{\pi}_{t-1} + \frac{1}{4} \hat{\pi}_{t-2} \right].$$

(43) shows that with no indexation, mean market real wage is a function of a distributed lag of past inflation rates with declining weights. The fact is there is no convincing evidence for such a result (see Altonji and Ashenfelter (1982), Ashenfelter and Card (1983), Stock and Watson (2007) for an opposite conclusion and suggestions of considering mean real wage a random walk) from which one concludes that some adjustment to inflation takes place which shortens the memory of the price setting process.

Before proceeding I note (43) offers an opportunity to comment on the large 1970's and 1980's literature on the behavior of wages. It was a virtually universal practice in the 1970's and 1980's to use as endogenous variable *the time rate of change of the real wage* and then use the first difference of output (ie. the growth rate) and inflation as explanatory variables (e.g. Tobin (1972) Eq. (2), surveys by Abraham and Haltwinger (1995), Brandolini (1995) and references there). Time change of the real wage is replaced by the left side of (43). Also, (43) is analogous to the wage equation in Tobin (1972) (who reviews wage equations in Hirsch (1972), de Menil and Enzler (1972) and Hymans (1972)) where the term \hat{w}_{t+1}^* is analogous to time change of productivity. The key difference is that in (43) the right hand side variable is not the first difference of inflation but a distributed lag of past inflation rates. It shows again that *if there is no indexation* the first difference of the real wage must vary with a distributed lag of inflation. Note that in (43) both the wage and inflation are endogenous. In some treatments this interdependence was removed by replacing inflation by expected inflation (e.g. Tobin (1972)) but many empirical studies disregard this point.

This discussion leads to two conclusions. First, distributed lag of inflation is absent from the empirical evidence on wage dynamics. Second, despite their empirical success, the key theoretical objection to the wage and price adjustment models of the 1970's and 1980's is their formulation as first difference since these imply that transitory disturbances may have permanent persistent effects (see Blanchard and Fischer (1989) page 544). To resolve this issue I avoid such formulation and reject the representation (43) as expressed in the preliminary formulation of (40a)- (40b), specified as an equation of the real wage rather than as a first difference of the real wage. I also reject the hypothesis of no short term indexation and turn now to discuss the alternatives.

5.4.2 *Mean Real Wage With Some Indexation*

The fact is that firms do provide some indexation of inflation, but in a complex manner. Some include in their nominal wage offers a premium for *expected* inflation which amounts to a fractional advanced indexation. Some empirical work appears to confirm this (e.g. Tobin (1972)) but it is hard to distinguish between actual date t inflation and date t expected inflation in t+1 since correlation between inflation rates in successive quarters is close to 1. A second practice is to offer a partial indexation through components of compensation such as cost sharing by the firm (i.e. rent paid by the firm) or benefits like medical insurance. A third practice commonly followed and established empirically (e.g. Cecchetti (1984)) is to change the frequency of wage adjustments with changes in the rate of inflation, which amounts to speeding up indexation with higher inflation rates.

I thus turn to the empirical evidence and here the picture is surprisingly clear in terms of estimates.

Notwithstanding the diverse models used to estimate wage dynamics, the empirical result that stands out is the effect of short term inflation on the real wage which has two parts. The first insists the inflation rate that matters is the current rate or current expected rate for next quarter but the practical difference between these is minor. Second, the elasticity of the real wage with respect to current inflation was estimated between 0.4 and 0.7 (e.g. Tobin (1972), Blanchard and Fischer (1989) Chapter 10). As explained earlier, one can think of this elasticity as an indirect expression of a short run Phillips Curve. This elasticity reflects the level of inflation risk sharing between a firm and its workers and results in a degree of indexation of the real wage. An elasticity of, say, 0.6 means that date t inflation rate of 1% reduces the equilibrium real wage in that period by 0.6% hence for one period the heavier weight is born by the workers. But this is only for one period. Lack of distributed lag effect means that the decline of the real wage takes place for one period and full indexation takes place after that period if inflation lasts for only one period. To have a persistent effect on the real wage the inflation rate has to exhibit persistence. However, since target inflation is incorporated into the wage setting process, this persistence of inflation away from target *must result from known and understood temporary forces* otherwise some agents may question their belief in a constant and known target and hence form their own belief about the actual target in the market. This is the assumption I adopt here for regular employees. For irregular employees I assume that inflation indexation takes place automatically since they are offered a current nominal wage which is fully indexed. There is no reasons to assume that this division of inflation risk applies to other societies or be the same in other periods of US history.

To interpret the empirical result above and translate the above assumption into the model, the assumed real wages of the groups at date t are then as follows

$$(44a) \quad \text{Group 1: real wage 1 date after offer} \quad W_t^{(t-1)} = \left(\frac{P_{t-1}}{P_t}\right)^v W_{t-1}^e$$

$$(44b) \quad \text{Group 2: real wage 2 dates after offer} \quad W_t^{(t-2)} = \left(\frac{P_{t-1}}{P_t}\right)^v W_{t-2}^e$$

$$(44c) \quad \text{Group 3: real wage 3 dates after offer} \quad W_t^{(t-3)} = \left(\frac{P_{t-1}}{P_t}\right)^v W_{t-3}^e$$

$$(44d) \quad \text{Group 4: real wage 4 dates after offer} \quad W_t^{(t-4)} = \left(\frac{P_{t-1}}{P_t}\right)^v W_{t-4}^e.$$

Hence the mean real wage of regular employees is now different from the reference wage and is defined by

$$W_t = \left(\frac{P_{t-1}}{P_t}\right)^v [W_{t-1}^e W_{(t-2)}^e W_{(t-3)}^e W_{(t-4)}^e]^{\frac{1}{4}} = \left(\frac{P_{t-1}}{P_t}\right)^v W_t^*.$$

Hence the real wages of regular and irregular employees are then, respectively

$$(45a) \quad W_t^R = \left(\frac{P_{t-1}}{P_t}\right)^v W_t^*$$

$$(45b) \quad W_t^{IR} = \left[\left(1 + \varrho\right) \frac{N_t}{L_t}\right]^\mu W_t^* e^{\rho_t^w}.$$

and mean real wage in the economy is

$$W_t = \left[\left(\frac{P_{t-1}}{P_t}\right)^v W_t^*\right]^{a_w} \left[\left(1 + \varrho\right) \frac{N_t}{L_t}\right]^\mu W_t^* e^{\rho_t^w} \right]^{(1-a_w)}$$

hence

$$W_t = W_t^* \left(\frac{P_t}{P_{t-1}}\right)^{-va_w} \left[\left(1 + \varrho\right) \frac{N_t}{L_t}\right]^{\mu(1-a_w)} e^{(1-a_w)\rho_t^w}.$$

The final wage is then specified by the system

$$(46) \quad \hat{w}_t = \hat{w}_t^* - \lambda_\pi \hat{\pi}_t - \lambda_u u_t, \quad u_t = \hat{q}_t - \hat{\pi}_t, \quad \lambda_\pi = va_w, \quad \lambda_u = \mu(1 - \lambda_w).$$

where

$$\hat{w}_t^* = B_w^\zeta (\hat{\zeta}_{t-1} + \hat{\zeta}_{t-2} + \hat{\zeta}_{t-3} + \hat{\zeta}_{t-4}) + \rho_t^* \quad , \quad B_w^\zeta = \frac{A_w^\zeta}{16} (\lambda_\zeta + \lambda_\zeta^2 + \lambda_\zeta^3 + \lambda_\zeta^4).$$

or

$$\hat{w}_{t+1}^* = \lambda_\zeta \hat{w}_t^* + B_w^\zeta [\hat{\zeta}_t + (1 - \lambda_\zeta) \hat{\zeta}_{t-1} + (1 - \lambda_\zeta) \hat{\zeta}_{t-2} + (1 - \lambda_\zeta) \hat{\zeta}_{t-3} - \lambda_\zeta \hat{\zeta}_{t-4}] + \rho_{t+1}^*.$$

Parameter values used in later chapters are those estimated empirically for the post war era: $\lambda_\pi = 0.6$, $\lambda_u = 0.15$.

With the approximate value of $a_w = 0.85$, they all imply $v = 0.71$, $\mu = 1$.

5.5 *Some Concluding Thoughts on Departures from the Wage in (49)*

The mean wage in (46), as a function of three factors, characterizes wage inflexibility. The contribution of the reference wage to (46) consists of smoothing out of future and past productivity thus averaging it over twice the offer's duration. Its dependence on the three variables that reflect productivity, inflation and unemployment is in line with results of the empirical literature but is also compatible in spirit with theoretical considerations underlying other approaches to wage inflexibility employed in work such as Shimer (2004), Hall (2005a), (2005b), Gertler and Trigari (2009), Blanchard and Galí (2010). Proposals to treat a wage rate either as a random walk (e.g. Altonji and Ashenfelter (1982), Stock and Watson (2007)) or as a constant (e.g. Hall (2005a)) are less persuasive. In short, this wage setting process is compatible both with known empirical results as well as recent conceptual developments. Some qualifications are, however, appropriate.

There is no doubt a careful student of labor markets will find time periods or countries in which the factors that contribute to wage setting are different from (46) or are the same but with different parameter values. For example, I have specifically argued that the firm and workers have stronger incentives to arrive at the

proposed solution the more real and human capital is invested in the production process. One can then conclude that the results of the theory would be bolstered by environments with heavy real and human capital investments but this points to one more factor that needs to be incorporated. In addition, one notes that in the last half century there were at least two deviations of mean wage setting from (46) for a significant length of time with important aggregate consequences, and I use Figure 3 to explain it.

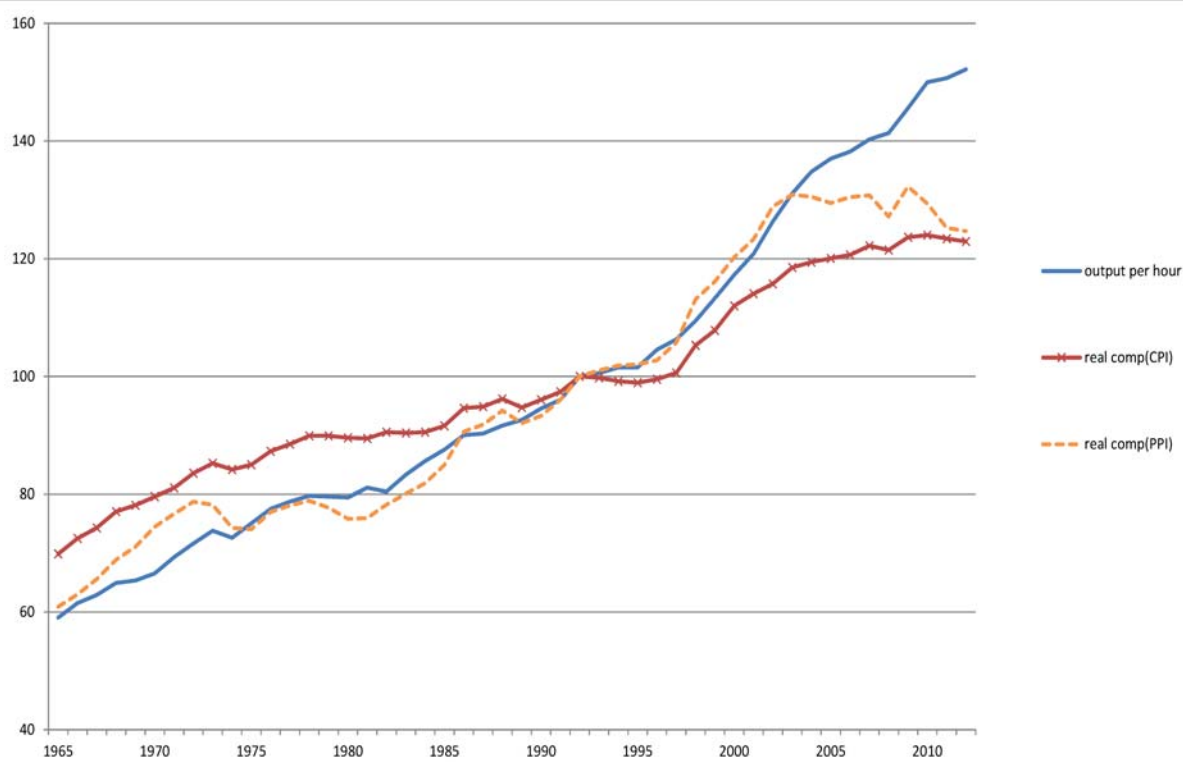


Figure 3: Output per man hour is for the US business sector from Table B-49 of the Economic Report of the President, 2014. Real Compensation (CPI) or (PPI) is total compensation per hour in the business sector, Table B-49 of the same report where the nominal total compensation is divided by the consumer price index or the producer price index for all finished good, Table B-65 of the same report. All indices are scaled at 1992 = 100

To discuss Figure 3 I first explain the data graphed. In addition to the wage, the relevant prices to a firm's decisions are prices of all other inputs since the firm's labor demand depends upon the *relative price of labor to other input prices*. Hence, the price index relevant to a firm is the Producer Price Index (PPI) not the Consumer Price Index (CPI), which is the index of interest to workers who make labor supply decisions based on the *relative price of labor to consumer goods*. The problem is that the PPI and the CPI evolve differently over time hence a firm and its workers disagree on what constitutes a relevant real wage. Hence, even if the firm and

the workers had a common desire to insure the real value of wages, there is a need to identify an index with which to define the real wage. The two different standard measures of real labor compensation per hour are graphed in Figure 3 and the three indices in Figure 3 are all normalized with a value of 100 in 1992. Since these are index numbers, one cannot state that one measure of compensation is larger than the other. Comparisons can be made relative to an arbitrary date when two series are normalized to a base of 100. The data in Figure 3 shows one can compare the real wage measured by total real compensation (PPI) with the real wage measured by total real compensation (CPI) relative to the values of baskets of goods in 1992. With this normalization one can draw two conclusions. First, during 1965-2012 the real wage (CPI) *has grown much slower than the real wage (PPI)*. Second, relative to 1992 values, the real wage (CPI) was lower than the real wage (PPI) in 1992-2012 and higher than the real wage (PPI) in 1965-1992. These comparisons are interesting and well known.

Much more significant comparisons can be made if the two series have a common trend (i.e. have the same mean growth rate) which is then estimated to verify that along this common trend the two data series match well. In that case one selects any date when the series match and normalize them to the same base of 100 at that date. All differences between the two series then reflect deviations *relative to the common trend*. Such comparison is possible between output per hour and real compensation (PPI). Figure 3 shows a normalization of the two series at 100 in 1992 places them very close to each other. Normalization of 100 in 1972 or in 2010 do not accomplish this desired goal. This means we can now compare real productivity per man hour (IPD) with total real compensation (PPI), *relative to their common trend*. With this interpretation in mind, the data then shows that between 1965 and 2012 there are two episodes when the real wage (PPI) deviated significantly from productivity per man hour (IPD). In 1967 -1974 the real wage (PPI) was significantly higher than hourly productivity (IPD) and in 2004-2012 the real wage (PPI) was significantly lower than hourly productivity (IPD). It is actually remarkable that although real wage (PPI) and real wage (CPI) have different trends, one can assert that relative to values in 1992 both real wage (PPI) and real wage (CPI) were higher than hourly productivity (IPD) in 1967 -1974 and both real wages were lower than hourly productivity (IPD) in 2003 -2012.

The fact that during 1966 -1974 the real wage with either (PPI) or (CPI) was higher than productivity (IPD) has important implications. First observe that although significant economic events took place during the years 1966 -1974, the unemployment rate fluctuated close to 5% but the rate of inflation began its historic rise. Hence, the wage setting model (46) would have fixed the wage *below* trend. It follows that in 1966 -1974 *mean real wage deviated from (46) and was too high relative to this norm*. Similarly, in 2004 -2012 *the mean real wage deviated from (46) and has been too low relative to this norm*. The large discrepancy during 1966 -1974

was the consequence of wages being pushed too high by the Viet-Nam war, the political environment where the US sought to eliminate poverty with the tools of the Great Society programs and by labor unions who were able to secure work benefits that emulated the 1949 Treaty of Detroit. These political forces were important but are not included in (46). The 2004 -2012 discrepancy begins with the massive elimination of medical insurance and retirement benefits by the private sector. It was continued during the Great Recession when compensations failed to keep pace with productivity. These developments have not been sufficiently studied as yet.

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